THRUST ENHANCEMENT THROUGH BUBBLE INJECTION INTO AN EXPANDING-CONTRACTING-NOZZLE WITH A THROAT

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ABSTRACT

This paper addresses the concept of thrust augmentation through bubble injection into an expanding-contracting nozzle with a throat. The presence of a throat in an expanding-contracting nozzle can result in flow transition from the subsonic regime to the supersonic regime (choked conditions) for a bubbly mixture flow, which may result in a substantial increase in jet thrust. This increase would primarily arise from the fact that the injected gas bubbles expand drastically in the supersonic region of the flow. In the current work, an analytical 1-D model is developed to capture choked bubbly flow in an expanding-contracting nozzle with a throat. The study provides analytical and numerical support to analytical observations and serves as a design tool for nozzle geometries which can achieve efficient choked bubbly flows through nozzles.

Starting from the 1-D mixture continuity and momentum equations along with an equation of state for the bubbly mixture, expressions for mixture velocity and gas volume fraction were derived. Starting with a fixed geometry and an imposed upstream pressure for a choked flow in the nozzle, the derived expressions were iteratively solved to obtain the exit pressures and velocities for different injected gas volume fractions. The variation of thrust enhancement with the injected gas volume fraction was also studied. Additionally, the geometric parameters were varied (area of the exit, area of the throat) to understand the influence of the nozzle geometry on the thrust enhancement and on the flow conditions at the inlet. This parametric study provides a performance map that can be used to design a bubble augmented waterjet propulsor, which can achieve and exploit supersonic flow. It was found that the optimum geometry for choked flows, unlike the optimum geometry under purely subsonic flows, had a dependence on the injected gas volume fraction. Furthermore, for the same injected gas volume fraction the optimum geometry for choked flows resulted in greater thrust enhancement compared to the optimum geometry for purely subsonic flows.

Keywords: bubble augmented waterjet, thrust augmentation, two-phase flow, bubbly mixture, speed of sound, waterjet propulsion

NOMENCLATURE

A : Cross-sectional area
A_{inlet} : Cross-sectional area of the inlet section
$A_{injection}$: Cross-sectional area of the injection section
A_{throat} : Cross-sectional area of the throat
A_{exit} : Cross-sectional area of the exit section
k : Polytropic gas constant
p : Pressure
p_{inlet} : Inlet/upstream pressure
p_{ref} : Reference pressure

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p_{inj^-} : Pressure immediately before the injection section

p_{inj^+} : Pressure immediately after the injection section

p_* : Throat pressure at choked condition

p_{exit} : Exit pressure

u_m : Mixture velocity

$u_{m,*}$: Mixture velocity at throat under choked condition

$u_{m,exit}$: Mixture velocity at exit

V_{inlet} : Inlet liquid velocity

V_{inj^+} : Velocity immediately after injection

V_{inj^-} : Velocity immediately before injection

α : Void fraction

α_0 : Injected void fraction

α_{ref} : Reference void fraction

α_{exit} : Exit void fraction

ρ_l : Liquid density

ρ_m : Mixture density

ξ_m : Normalized thrust parameter

$\xi_{m,sub}$: Normalized thrust parameter (purely subsonic flow)

INTRODUCTION

Several studies [1,2] have indicated that bubble injection in a waterjet can significantly improve the net thrust. The importance of this concept was attributed to the fact that bubble augmented thrust can be achieved even at very high vehicle speeds unlike traditional propulsion devices.

Various analytical, numerical and experimental efforts have been carried out since to formalize the concept of bubble augmented thrust [3,4], and a number of prototypes have been built for experimental verification of this concept [5,6]. In most of these experimental set-ups, the fluid enters a ramjet where it is compressed first by passing through a diffuser (ram effect), then pressurized gas is injected into the fluid via mixing ports, which acts like the energy source of the ramjet. The multiphase mixture is then accelerated through a converging nozzle. To obtain the section averaged mixture flow field inside the air augmented nozzle numerically, one can use classical quasi-one dimensional analyses [7-11]. In such approaches, flow quantities are assumed to be uniform in a direction perpendicular to the axis of the nozzle (i.e. only quantities averaged over a cross-section are used). Furthermore, in these approaches, the mixture flow through the nozzle is always assumed to be purely subsonic.

Recent numerical and experimental investigations [12-14] have demonstrated large thrust enhancement for high void fraction, purely subsonic flows through an expanding-contracting nozzle, the Bubble Augmented Propulsor (BAP) (Figure 1). Studies have also been conducted to establish the optimum geometry of an expanding-contracting nozzle for maximum thrust enhancement under purely subsonic flow conditions [13]. While simulations and experimental investigations have demonstrated thrust augmentation in supersonic mixer-ejector systems [15,-17], for gas-water systems (BAP), less information is available in the literature regarding the flow behavior of a bubbly mixture through an expanding-contracting nozzle under choked conditions. There is some evidence suggesting that the presence of a throat in an expanding-contracting nozzle can result in flow transition from the subsonic regime to the supersonic regime (choked conditions) and thereby result in a substantial increase in jet thrust [18]. This increase primarily arises from further expansion of the injected gas bubbles in the supersonic region of the flow (region after the throat).

The current study considers a 1-D analytical model to predict the thrust characteristics of an expanding-contracting nozzle under choked conditions. The study uses an analytical model to optimize the geometry of the expanding-contracting nozzle for maximum thrust enhancement under choked conditions taking into account the effect of the injected void fraction and the effect of the upstream pressure in the nozzle.

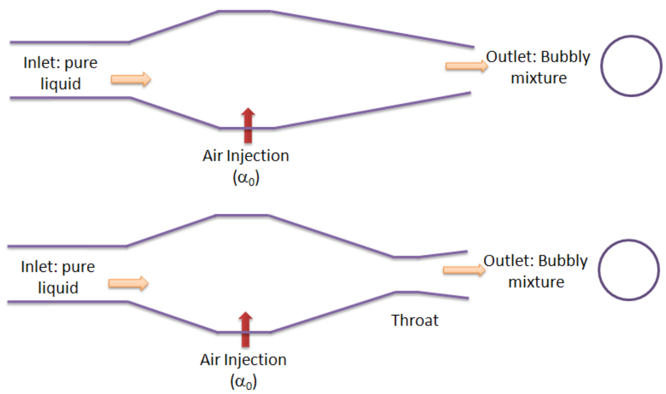

Figure 1. Expanding-contracting nozzles : Absence of throat implies purely subsonic flow (top), Presence of throat allows for the possibility of choked flow (bottom).

MODELING OF CHOKED FLOW

When bubbles are injected continually and steady state conditions are achieved, the 1-D model described above reduces to the following form:

$$\frac{\partial(\rho_m u_m A)}{\partial x} = 0, \tag{1}$$

$$\frac{1}{A}\frac{\partial(\rho_m u_m A u_m)}{\partial x} + \frac{\partial p}{\partial x} = 0. \tag{2}$$

Where A is the cross-sectional area of the nozzle at location x, u_m is the mixture velocity, ρ_m is the mixture density and p is the pressure. By neglecting the density of the injected air relative to the liquid density, ρ_l, the mixture density, ρ_m, reduces to

$$\rho_m = \rho_l\left(1-\alpha\right). \tag{3}$$

The void fraction , α, can be connected to the pressure through an equation of state for the mixture, which depends on the gas polytropic compression law constant k, as follows [19]:

$$\frac{p}{p_{ref}} = \left[\frac{\alpha_{ref}\left(1-\alpha\right)}{\alpha\left(1-\alpha_{ref}\right)}\right]^k, \tag{4}$$

where p_{ref} is the reference pressure corresponding to a reference void fraction of α_{ref}. Here k can be taken to be 1 (isothermal conditions) since the gas bubbles execute relatively slow and small amplitude oscillations.

The momentum equation (2) can be expanded and rewritten in the following form:

$$\frac{1}{A}\frac{dA}{dx}u_m^2\rho_m + u_m^2\frac{d\rho_m}{dx} + 2\rho_m u_m\frac{du_m}{dx} = -\frac{dp}{dx}, \tag{5}$$

which after use of the continuity equation (1), simplifies to:

$$\rho_m u_m \frac{du_m}{dx} = -\frac{dp}{dx}. \tag{6}$$

By combining (5) and (6) and rearranging, we obtain the following expression:

$$\frac{1}{A}\frac{dA}{dx} = \frac{1}{u_m^2\rho_m}\frac{dp}{dx} - \frac{1}{\rho_m}\frac{d\rho_m}{dx}. \tag{7}$$

Using the expression for the sound speed of the mixture,

$$\begin{aligned} c_m^2 &= \frac{dp}{d\rho_m}, \\ c_m^2 &= \frac{kp_{ref}}{\rho_l}\frac{(1-\alpha)^{k-1}}{\alpha^{k+1}}\frac{\alpha_{ref}^k}{(1-\alpha_{ref})^k}, \qquad k \neq 1, \\ &= \frac{p_{ref}}{\rho_l}\frac{1}{\alpha^2}\frac{\alpha_{ref}}{(1-\alpha_{ref})}, \qquad k = 1. \end{aligned} \tag{8}$$

Equation (7) gives the following relation connecting the area gradient to the pressure gradient within the 1-D nozzle:

$$\frac{1}{A}\frac{dA}{dx} = \frac{1}{\rho_m}\frac{dp}{dx}\left(\frac{1}{u_m^2} - \frac{1}{c_m^2}\right). \tag{9}$$

From Equation (9), one can see that if the nozzle geometry is such that $dA/dx = 0$ at a given location, then either a zero pressure gradient occurs at that location, $dp/dx = 0$, or the liquid speed equals the sound speed of the medium, $u_m = c_m$, (choked flow). Therefore, if a throat is present in the nozzle, the mixture speed can reach there the sound speed and the flow can transition from a subsonic regime (upstream of the throat) to a supersonic regime (downstream of the throat).

For a typical nozzle designed for thrust enhancement by bubble injection as sketched in Figure 2, if choked flow occurs then the physical space may be broken down into three regions:

- Liquid flow through a diverging nozzle,
- Flow across the injection section ,
- Bubbly choked flow after the injection section.

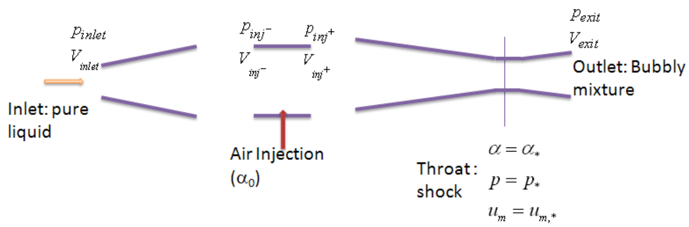

Figure 2: Typical nozzle design for thrust enhancement by bubble injection.

For the region with bubbly flow (downstream of the injection), Equation (2) can be integrated accounting for (3) and (4) to obtain the mixture velocities in the nozzle as functions of the local gas volume fraction and the reference quantities ($u_{m,ref}$ is the mixture velocity where the pressure is p_{ref} and the void fraction is α_{ref})

$$u_m^2 - u_{m,ref}^2 = \frac{2kp_{ref}\alpha_{ref}}{\rho_l(1-\alpha_{ref})}\left[\left(\frac{1-\alpha_{ref}}{\alpha_{ref}}\right)-\left(\frac{1-\alpha}{\alpha}\right)+\ln\left(\frac{(1-\alpha_{ref})\alpha}{\alpha_{ref}(1-\alpha)}\right)\right]. \tag{10}$$

Of the two solutions in (10), one solution corresponds to subsonic flow and the other to supersonic flow. Using the continuity equation (1), the reference velocity in the above equation can be expressed in terms of a reference area, A_{ref} , and the reference void fraction to arrive at the following simplification:

$$u_m^2\left[1-\left(\frac{(1-\alpha)A}{(1-\alpha_{ref})A_{ref}}\right)^2\right] = \frac{2kp_{ref}\alpha_{ref}}{\rho_l(1-\alpha_{ref})}\left[\left(\frac{1-\alpha_{ref}}{\alpha_{ref}}\right)-\left(\frac{1-\alpha}{\alpha}\right)+\ln\left(\frac{(1-\alpha_{ref})\alpha}{\alpha_{ref}(1-\alpha)}\right)\right]. \tag{11}$$

Under choked conditions, the void fraction at the throat, α_*, can be computed by equating the mixture speed, u_m in (11), to the sound speed, c_m in (8). This gives:

$$\frac{1}{2\alpha_*^2} = \left[1-\left(\frac{(1-\alpha_*)A_{throat}}{(1-\alpha_{ref})A_{ref}}\right)^2\right]^{-1} \times \left[\frac{1}{\alpha_{ref}}-\frac{1}{\alpha_*}+\ln\left(\frac{(1-\alpha_{ref})\alpha_*}{(1-\alpha_*)\alpha_{ref}}\right)\right]. \tag{12}$$

A_{throat} in the above relation is the throat area. In the current study, for the sake of convenience, the BAP section immediately after injection is used to define the reference quantities, i.e. :

$$\begin{aligned} \alpha_{ref} &= \alpha_0 \\ A_{ref} &= A_{injection} \\ p_{ref} &= p_{inj^+} \\ u_{m,ref} &= V_{inj^+} \end{aligned}, \tag{13}$$

where α_0 is the injected void fraction, $A_{injection}$ is the area of the injection section, p_{inj^+} is the pressure immediately after injection and V_{inj^+} is the velocity immediately after injection. Given the geometry and the injected void fraction, the throat void fraction is obtained by iteratively solving equation (12). Once the throat void fraction is known, the mixture velocity at the throat, $u_{m,*}$, which is equal to the mixture sound speed at the throat, can be obtained using (8):

$$u_{m,*} = \left(\frac{p_{inj^+}}{\rho_l} \frac{1}{\alpha_*^2} \frac{\alpha_{ref}}{\left(1-\alpha_{ref}\right)} \right)^{1/2} . \tag{14}$$

Even though the reference pressure, p_{inj^+}, and reference velocity, V_{inj^+}, cannot be evaluated independently with the available information at this stage, the following non-dimensional parameter (at the reference section) can be evaluated using equations (1) and (14):

$$\begin{aligned} \mathrm{K} &= \frac{\rho_l (1-\alpha_0) V_{inj^+}^2}{2 p_{inj^+} \alpha_0} \\ &= \frac{\rho_l (1-\alpha_0) u_{m,*}^2}{2 p_{inj^+} \alpha_0} \left[\frac{(1-\alpha_*) A_{throat}}{(1-\alpha_0) A_{injection}} \right]^2 \\ &= \frac{1}{2\alpha_*^2} \left[\frac{(1-\alpha_*) A_{throat}}{(1-\alpha_0) A_{injection}} \right]^2 . \end{aligned} \tag{15}$$

The above non-dimensional parameter is crucial for relating the specified inlet pressure (p_{inlet}) to the pressure at the reference section. Using the continuity and momentum equations, the velocity, V_{inj^-}, and pressure, p_{inj^-}, at the section immediately before injection can be related to the corresponding quantities after injection and the injected void fraction, α_0, by the following relationships [13]:

$$V_{inj^-} = \left(1-\alpha_o\right) V_{inj^+} . \tag{16}$$

$$p_{inj^-} = p_{inj^+} + \rho_l V_{inj^+}^2 \alpha_o \left(1-\alpha_o\right). \tag{17}$$

Similarly, the inlet velocity, V_{inlet}, and the inlet pressure, p_{inlet}, can be related to the corresponding quantities immediately before injection [13]:

$$A_{inlet} V_{inlet} = A_{injection} V_{inj^-} \ , \tag{18}$$

$$p_{inlet} + \frac{\rho_l}{2} V_{inlet}^2 = p_{inj^-} + \frac{\rho_l}{2} V_{inj^-}^2 \ . \tag{19}$$

In the above relations, A_{inlet} is the area of the inlet section. Using equations (16) and (18), the inlet velocity can be expressed in terms of the reference velocity, V_{inj^+}, as follows:

$$V_{inlet} = \left[\frac{A_{injection} (1-\alpha_0)}{A_{inlet}} \right] V_{inj^+} . \tag{20}$$

Expressing V_{inlet} in equation (19) using equation (20) and by expressing the right hand side of equation (19) in terms of V_{inj^+} and p_{inj^+} (using equation (16) and (17)), we have :

$$p_{inlet} + \frac{\rho_l}{2}\left[\frac{A_{injection}(1-\alpha_0)}{A_{inlet}}\right]^2 V_{inj^+}^2 = p_{inj^+} + \rho_l V_{inj^+}^2 \alpha_o \left(1-\alpha_o\right) + \frac{\rho_l}{2}(1-\alpha_0)^2 V_{inj+}^2. \qquad (21)$$

Upon simplification, the above equation takes the following form:

$$p_{inj^+} = p_{inlet}\left[1 + \frac{\rho_l V_{inj^+}^2 (1-\alpha_o)}{2 p_{inj^+}}\left[\alpha_o + 1 - \left(\frac{A_{injection}}{A_{inlet}}\right)^2 (1-\alpha_o)\right]\right]^{-1}. \qquad (22)$$

Using equation (15) in equation (22), we obtain:

$$p_{inj^+} = p_{inlet}\left[1 + \mathrm{K}\alpha_o\left[\alpha_o + 1 - \left(\frac{A_{injection}}{A_{inlet}}\right)^2 (1-\alpha_o)\right]\right]^{-1}. \qquad (23)$$

The above relation lets us determine the reference pressure for a given upstream pressure. Once the reference pressure is known, equation (15) can be used to obtain the reference velocity, V_{inj^+}. Once the reference values are known, the inlet velocity V_{inlet} can be determined using equation (20). The V_{inlet} computed using the above procedure is the minimum inlet velocity needed for obtaining choked flow in the nozzle. For velocities less than V_{inlet}, the flow should remain purely subsonic.

Flow quantities at all locations between the inlet and the region just after injection can then be obtained through simple application of the continuity and the momentum equations. Once the throat void fraction, α_*, and the reference pressure, p_{inj^+}, are known, the pressure at the throat, p_* can be obtained using the equation of state –(4). To obtain the flow quantities between the section after injection and the exit, an inverse technique is employed. Instead of starting with the area of a location along the axis of a nozzle and computing the flow quantities, in the inverse technique, one starts with a given ratio of p/p_* and computes the corresponding area of the section and therefore its location along the axis of the nozzle. This technique is ideally suited for obtaining choked flow solutions where a given ratio of A/A_{throat} can have one flow solution corresponding to the subsonic region ($p/p_* > 1,\ \alpha/\alpha_* < 1,\ u_m/u_{m,*} < 1$) and another one corresponding to the supersonic region ($p/p_* < 1,\ \alpha/\alpha_* > 1,\ u_m/u_{m,*} > 1$). The ratio p/p_* is varied from a very large value to a very small value. For each ratio of p/p_*, equation (4) determines α/α_*, equation (10) determines $u_m/u_{m,*}$ and equation (1) determines A/A_{throat}. For $p/p_* > 1$, all solutions are recorded for $1 < \frac{A}{A_{throat}} \le \frac{A_{injection}}{A_{throat}}$ - this corresponds to the subsonic solution of the flow between the injection section and the throat. For $p/p_* < 1$, all solutions are recorded for $1 < \frac{A}{A_{throat}} \le \frac{A_{exit}}{A_{throat}}$ (A_{exit} being the area of the exit section)- this corresponds to the supersonic solution of the flow between the throat and the exit. The solution corresponding to $A = A_{exit}$ gives us the exit pressure, p_{exit}, the exit mixture velocity, $u_{m,exit}$, and the exit void fraction, α_{exit}.

Normalized Thrust Increase

The thrust of the nozzle can be given as follows:

$$T_{R,\alpha} = A_{exit}\left[p_{exit} + \rho_l\left(1-\alpha_{exit}\right)u_{m,exit}^2\right] - A_{inlet}\left[p_{inlet} + \rho_l V_{inlet}^2\right]. \qquad (24)$$

The normalized thrust gain parameter is defined as the difference between the thrust with bubble injection and the thrust without bubbles injection, divided by the inlet liquid momentum rate:

$$\xi_m = \frac{T_{R,\alpha} - T_{R,0}}{T_{m-inlet}} = \frac{T_{R,\alpha} - T_{R,0}}{\rho_l A_{inlet} V_{inlet}^2} . \tag{25}$$

$T_{R,0}$ in the above relation is the nozzle thrust with zero bubble injection, $T_{m-inlet}$ is the inlet momentum rate.

It can be shown [13] that for purely subsonic flow, the normalized thrust parameter can be given by the following analytical expression:

$$\xi_{m,sub} = \frac{1}{2}\left(\frac{A_{exit}}{A_{inlet}}\right)^{-2}\left[1 - \left(\frac{(1-\alpha_0)}{(1-\alpha_{exit})} + \alpha_o \left(\frac{A_{exit}}{A_{injection}}\right)^2\right)\frac{1}{(1-\alpha_0)}\right] + \left(\frac{A_{exit}}{A_{inlet}}\right)^{-1}\left(\frac{\alpha_{exit}}{1-\alpha_{exit}}\right). \tag{26}$$

RESULTS AND DISCUSSION

Choked flow versus purely subsonic flow

The BAP geometry with a throat shown in Figure 3 is used to demonstrate a choked flow computation. The geometrical conditions A_{throat}/A_{inlet}=1.06, A_{exit}/A_{inlet}=1.4, $A_{injection}/A_{inlet}$=3.4, an upstream pressure, p_{inlet}= 5 atm, and $\alpha_0 = 0.2$ are used for the computations.

Figure 4 through Figure 6 show the axial distributions of flow quantities in the nozzle. Under choked conditions, the flow makes a transition from the subsonic regime to the supersonic regime across the throat where the mixture velocity becomes equal to the mixture sound speed at the throat (about 30 m/s for these conditions). This can be seen clearly in Figure 5. Because of this transition, in the section after the throat, the void fraction and the mixture velocity continue to *increase* (contrary to the subsonic case) and the pressure continues to decrease even though the cross-sectional area of the nozzle increases. This, in turn, results in very high exit velocities and thereby very high momentum thrust.

If the flow had remained subsonic (as illustrated in the figures), the presence of a throat would merely increase the flow velocity and decrease the pressure in the converging section and then bring them back to the previous state in the expansion section. Thus in purely subsonic flows, the throat does not provide any gain in thrust. To obtain the subsonic flow solution for the mass flow rate (or inlet velocity) predicted by the choked flow solution, the continuity, momentum and the equation of state for the mixture flow are simultaneously solved while imposing the outside ambient pressure at the nozzle exit [14].

Figure 7 shows contour plots of the pressure, velocity and void fraction under choked conditions, along the length of the nozzle.

The normalized thrust gain parameter for choked flow (ξ_m) for this configuration is 0.80 which is much higher than the normalized thrust gain parameter for subsonic flow under this configuration ($\xi_{m,sub}$ =0.013).

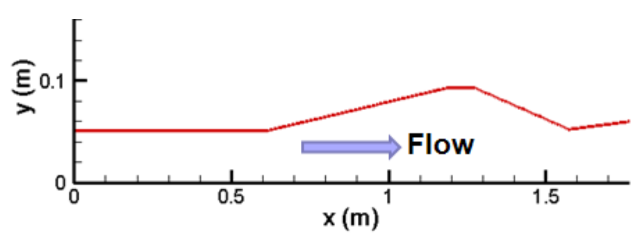

Figure 3: Geometry of an expanding-contracting nozzle incorporating a throat.

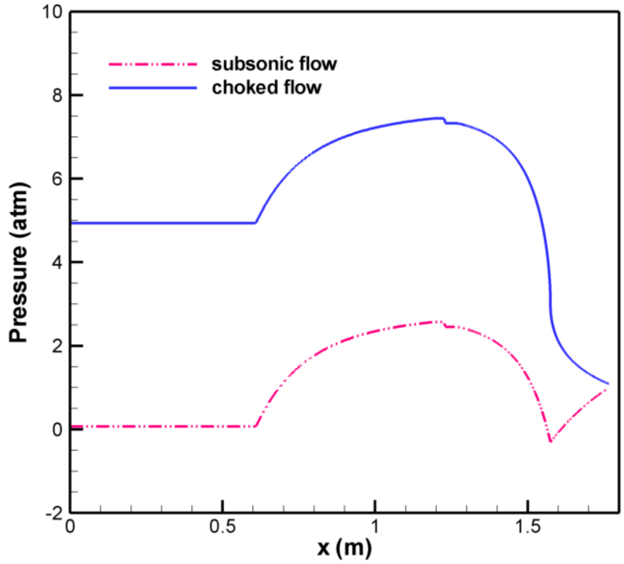

Figure 4: Axial distribution of the pressure, p, from the inlet to the exit. Curves for choked flow versus purely subsonic flow are shown.

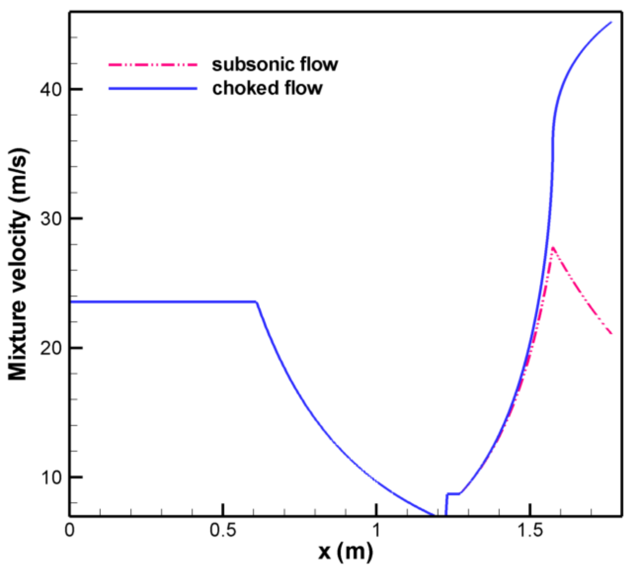

Figure 5: Axial distribution of mixture velocity from the inlet to the exit for the choked flow versus the purely subsonic flow.

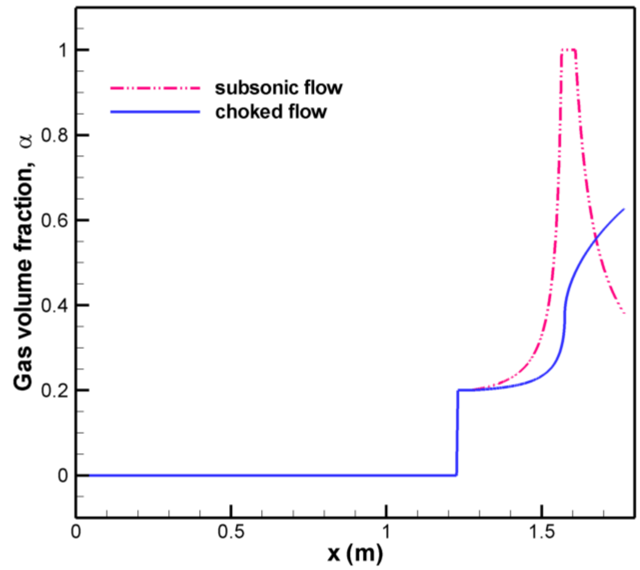

Figure 6: Axial distribution of gas volume fraction, α***, from the inlet to the exit. The void fraction of the supposed subsonic flow near the throat results in phase separation in the segment (1.5<x<1.65) of the BAP nozzle.***

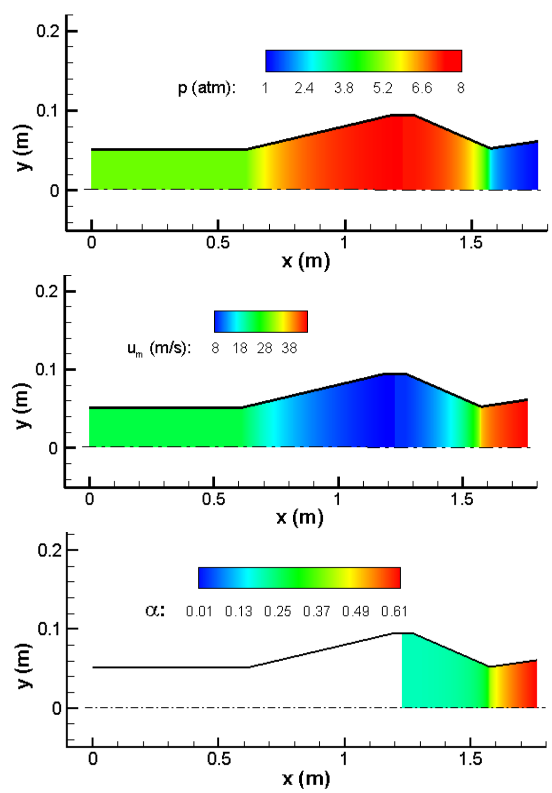

Figure 7: Contour plots of axial distribution of flow quantities in the BAP nozzle: pressure (top), velocity (middle), void fraction (bottom).

Optimum Configuration of the Expanding-Contracting nozzle: Effect of A_{throat}/A_{inlet} and A_{exit}/A_{inlet}

Since all the geometric parameters in the governing equations appear as ratios, all the flow quantities along the nozzle would be the same for any A_{inlet} as long as the geometric ratios $A_{injection}/A_{inlet}$, A_{throat}/A_{inlet}, A_{exit}/A_{inlet} are preserved.

The governing equations indicate that the throat area and the exit area are two of the most crucial parameters that decide the throat flow quantities and subsequently the normalized thrust gain of the BAP under choked conditions. The current section tries to identify the best A_{throat}/A_{inlet} and A_{exit}/A_{inlet} ratios for optimum thrust enhancement for a given upstream pressure, p_{inlet}, of 5 atm and $A_{injection}/A_{inlet}$=3.4. From a practical standpoint, the inlet velocity, V_{inlet}, into the nozzle required for achieving choked flow is a very important parameter. It is important that the inlet velocity, which cannot be increased any further once the nozzle flow is choked, is a quantity that can be achieved in a laboratory setting using conventional pumps.

Figure 8, Figure 9 and Figure 10 show performance maps of ξ_m with varying A_{throat}/A_{inlet} and A_{exit}/A_{inlet} for $\alpha_0 = 0.2$, $\alpha_0 = 0.4$ and $\alpha_0 = 0.6$ respectively. Drawn on these surface plots are the contour plots of ξ_m. From the figures, it appears that ξ_m continues to increase with increasing A_{exit}/A_{inlet} when the injected void fraction is relatively low (0.2 and 0.4). However, the rate of increase of ξ_m with increase in A_{exit}/A_{inlet} somewhat saturates when A_{exit}/A_{inlet} is about 6. The ξ_m for $A_{exit}/A_{inlet} = 6$ is about 90% of the ξ_m for $A_{exit}/A_{inlet} = 10$. For an injected void fraction of 0.6, ξ_m has a real optimum at $A_{exit}/A_{inlet} = 3.1$. A_{throat}/A_{inlet} has an optimum close to 2 for all the cases (Table 1).

It must be noted that in contrast to choked flow regimes, the purely subsonic flow regimes produce an optimum $\xi_{m,sub}$ for $A_{exit}/A_{inlet} = 1$ [13] irrespective of the injected void fraction, α_0. Table 2 shows the values of optimum $\xi_{m,sub}$ for different injected void fractions. Comparing values between Table 1 and Table 2, one can see that under the choked flow conditions the nozzle produces greater thrust gain than under purely subsonic conditions. Also, the optimum value of both ξ_m and $\xi_{m,sub}$ increase with increasing injected void fraction.

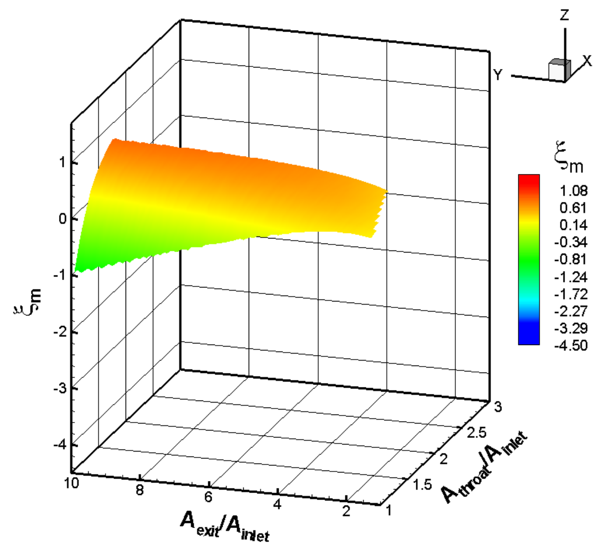

Figure 8: Performance map of ξ_m ***with varying*** A_{throat}/A_{inlet} ***and*** A_{exit}/A_{inlet} ***(*** $\alpha_0 = 0.2$***,*** p_{inlet} ***=5 atm).***

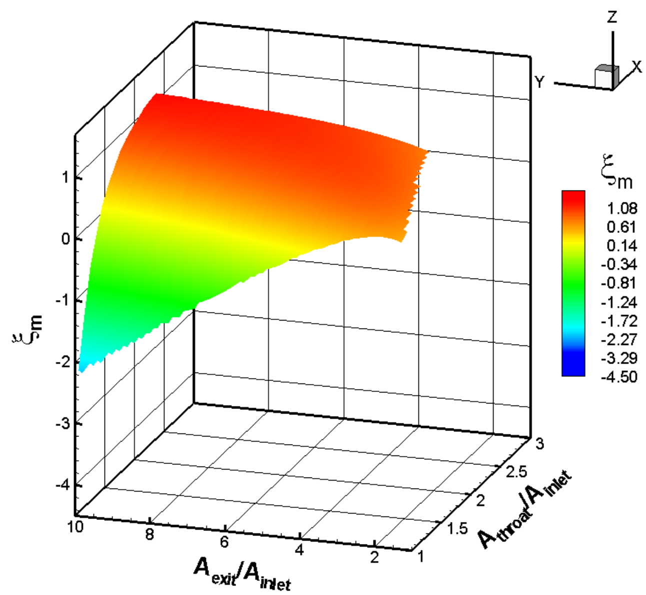

Figure 9: Performance map of ξ_m ***with varying*** A_{throat}/A_{inlet} ***and*** A_{exit}/A_{inlet} ***(*** $\alpha_0 = 0.4$***,*** p_{inlet} ***=5 atm).***

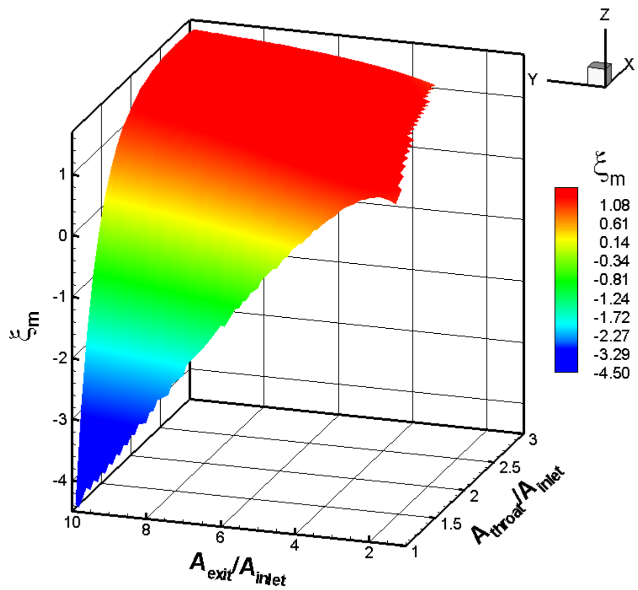

Figure 10: Performance map of ξ_m with varying A_{throat}/A_{inlet} and A_{exit}/A_{inlet} ($\alpha_0 = 0.6$, p_{inlet} =5 atm).

α_0	ξ_m *(optimum)*	A_{throat}/A_{inlet} *(optimum)*	A_{exit}/A_{inlet} *(optimum)*
0.2	0.81	1.7	10.0
0.4	1.20	2.3	10.0
0.6	1.68	1.6	3.1

Table 1: Optimum Geometric ratios for different injected void fractions under choked conditions.

α_0	$\xi_{m,sub}$ *(optimum)*	A_{exit}/A_{inlet} *(optimum)*
0.2	0.13	1.0
0.4	0.33	1.0
0.6	0.75	1.0

Table 2: Optimum Geometric ratios for different injected void fractions under subsonic conditions.

Figure 11 shows a map of the inlet velocity (V_{inlet}) with varying A_{throat}/A_{inlet} and A_{exit}/A_{inlet}. It can be seen from the figure that the inlet velocity does not change with changing exit area and it increases rapidly with increasing throat area. This is consistent with the facts that a higher mass flow rate is needed to achieve a choked flow for a larger throat area and that the exit area has no influence on the upstream flow quantities.

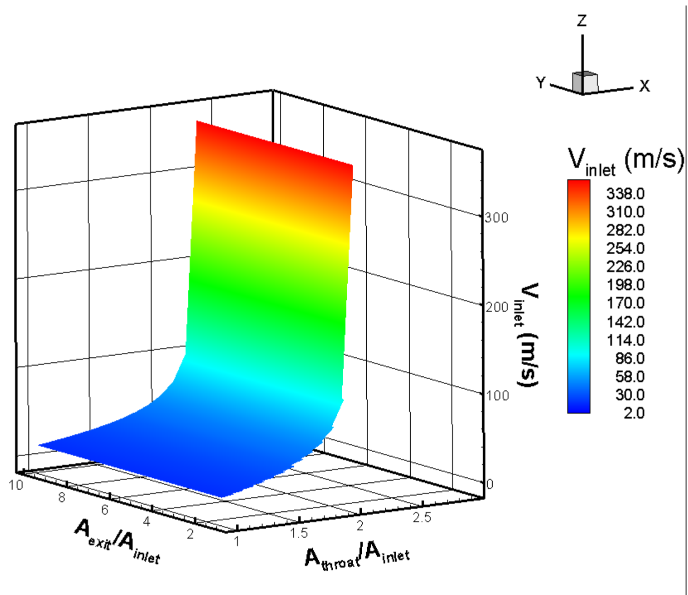

Figure 11: Inlet velocity map with varying A_{throat}/A_{inlet} and A_{exit}/A_{inlet} ($\alpha_0 = 0.4$, p_{inlet} =5 atm).

Effect of p_{inlet} and $A_{injection}/A_{inlet}$ on the optimum normalized thrust gain parameter

In the previous section, all the results presented were for a constant value of p_{inlet} (5 atm) and $A_{injection}/A_{inlet}$ (3.4). This section investigates the effect of changing these quantities on the normalized thrust gain parameter.

Figure 12 shows a plot of the optimum ξ_m versus α_0 for different inlet/upstream pressures. It is quite clear from the figure that the optimum values of ξ_m depend only on the geometric parameters and the injected void fraction and do not change with changing p_{inlet}.

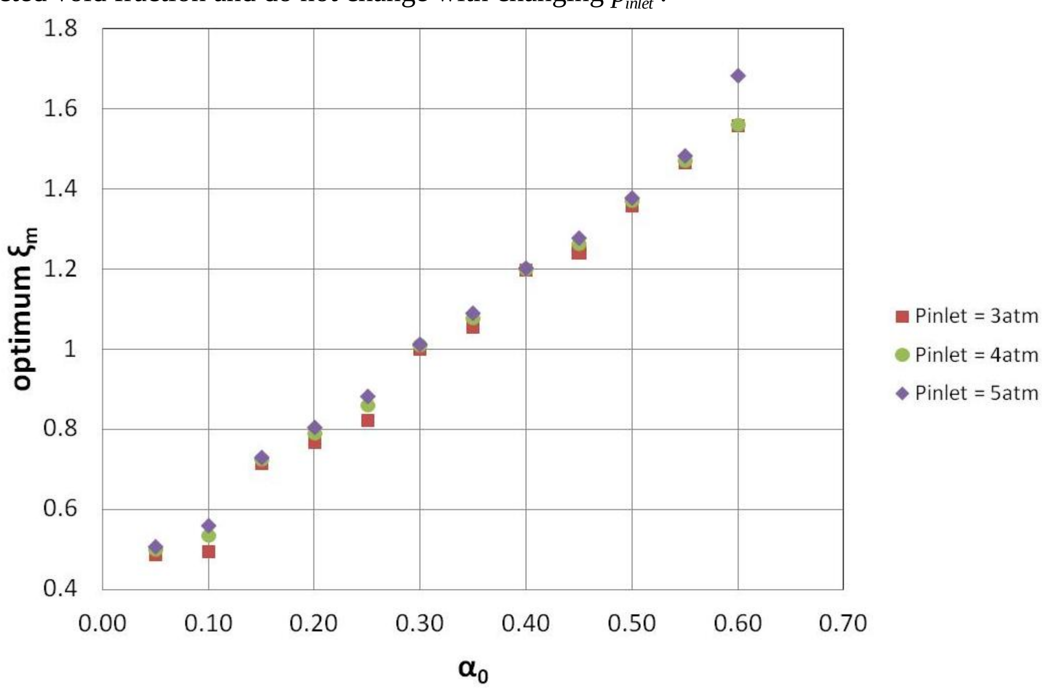

Figure 12: Optimum ξ_m versus α_0 for different inlet/upstream pressures.

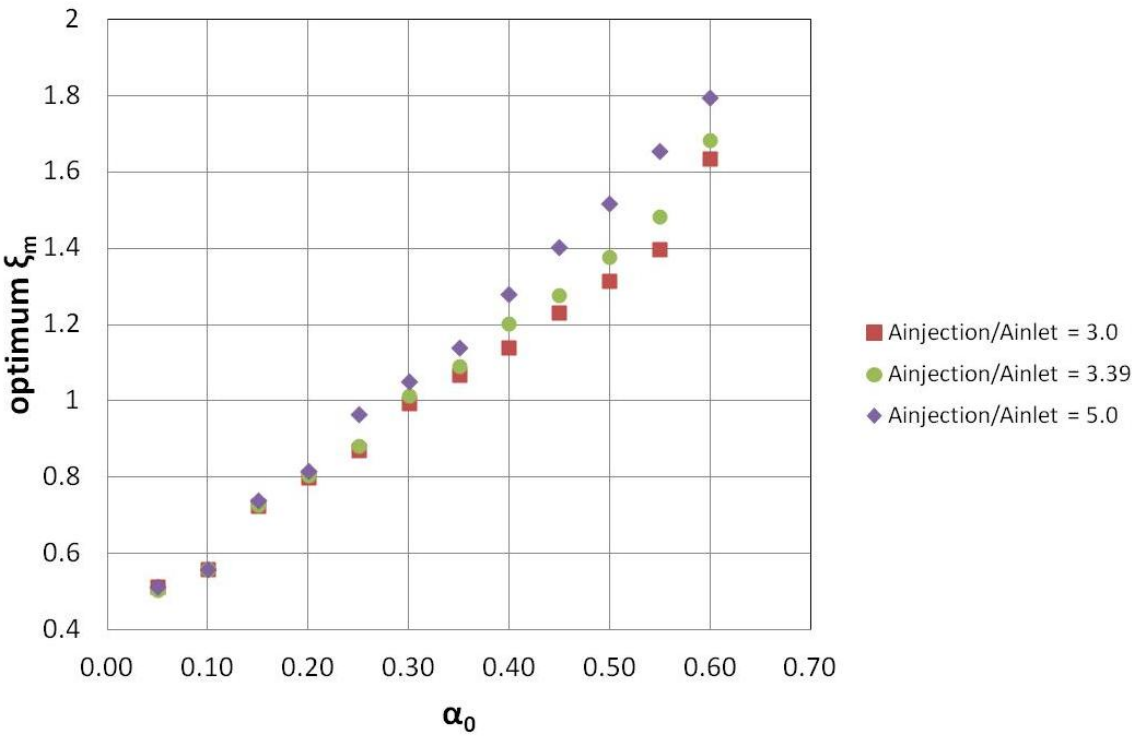

Figure 13: Optimum ξ_m ***versus*** α_0 ***for different ratios of*** $A_{injection} / A_{inlet}$.

Figure 13 shows a plot of the optimum ξ_m versus α_0 for different ratios of $A_{injection} / A_{inlet}$. It appears from the figure that the optimum ξ_m only weakly depends on the ratio $A_{injection} / A_{inlet}$. It must be noted that each point on the curves plotted on Figure 12 and Figure 13 is obtained using a performance map (similar to the ones on Figure 8, Figure 9 and Figure 10) for the corresponding conditions. A slight variation in the discretization resolution of these maps results in the wiggly nature of the curves on these figures.

CONCLUSION

One of the primary objectives of this study was to optimize the geometry of an expanding-contracting nozzle in order to obtain high thrust enhancement under choked conditions. The optimum normalized thrust gain was sought as a function of the area ratios A_{exit} / A_{inlet}, A_{throat} / A_{inlet}, and the injected void fraction. The optimum thrust gain showed little dependence on the injection area ratio, $A_{injection} / A_{inlet}$, and the inlet (upstream) pressure. It was found that the inlet velocity required for a choked flow did not depend on the exit area but increased rapidly with increasing throat area. Even though the optimum thrust gain occurs for a relatively large throat area, the corresponding values of the inlet velocity suggest that an engineering compromise would be necessary to use a nozzle design that has a fairly good thrust gain but also a practical inlet velocity V_{inlet}. For a given injected gas volume fraction, the optimum geometry for choked flow (i.e. supersonic exit flow) resulted in a high thrust gain compared to the optimum geometry for purely subsonic flows. This study provided a nozzle design, which was built and is presently being tested experimentally at DYNAFLOW to study the phenomenon of choked bubbly flow in nozzles.

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