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Homotopy analysis solution of free convection flow on a horizontal impermeable surface embedded in a saturated porous medium

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ABSTRACT

In this paper, the analytic solution of the buoyancy-driven flow over a horizontal impermeable flat plate embedded in a saturated porous medium is derived using the newly developed analytic method, namely the homotopy analysis method (HAM). The HAM results show great agreement comparing with numerical results. HAM contains an auxiliary parameter $\hbar$ that provides a simple way of controlling and adjusting the convergence region. The resultant analytic solution is valid for all acceptable values of the temperature exponent parameter λ.

1. Introduction

The prediction of convection through porous medium has numerous applications in geothermal-reservoir engineering [1–3], thermal insulation engineering [4], cooling of electronic systems, porous journal bearings, petroleum recovery, filtration processes, ceramic processing and ground water pollution analysis. For example, in modeling the geothermal power plants, the problem can be idealized as the rate of heat transfer from a heated or cooled horizontal flat plate embedded in a saturated porous medium as is the case in the present literature. It has been recognized for some time that convection in a saturated porous medium and in an incompressible fluid has much in common [5]. Thus, in analogous to the classical free convection problems at high Grashof numbers treated by Stewartson [6] and Gill [7], it can be assumed that convective heat transfer in a porous medium at high Rayleigh numbers takes place in a thin layer adjacent to the heated or cooled surfaces [5]. Recent finite difference solutions by Cheng et al. [8] for free convection in a liquid-dominated geothermal reservoir show that boundary layer behavior becomes increasingly pronounced in the flow field near the heated or cooled surfaces as the Rayleigh number of the reservoir is increased. The boundary layer approximations have been employed earlier by Wooding [9] to obtain analytical solutions to a number of free convection problems in a saturated porous medium at high Rayleigh numbers.

In this work, the approximate analytic solution of free convective flow over a horizontal impermeable surface embedded in a saturated porous medium by homotopy analysis method is investigated. As it will be shown in the next section, the governing non-linear partial differential equations transform into a coupled pair of non-linear ordinary differential equations with two-point boundary conditions through similarity solution, then this coupled pair of non-linear ODEs will be solved using HAM.

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Nomenclature

A	constant defined by Eq. (6)
C	specific heat of the convective fluid
f	dimensionless stream function
g	acceleration due to gravity
h	local heat transfer coefficient
$\hbar$	auxiliary parameter
HAM	homotopy analysis method
H	auxiliary function
K	permeability of the porous medium
k	thermal conductivity of the porous medium
$\mathscr{L}$	linear operator of HAM
$\mathscr{N}$	non-linear operator
P	pressure
T	temperature
u	velocity component in r-direction
w	velocity component in z-direction

Greek symbols

α	equivalent thermal diffusivity
β	constant, volumetric thermal expansion coefficient (1/K)
η	dimensionless similarity variable
θ	dimensionless temperature
λ	temperature exponent parameter
μ	viscosity of convective fluid
ρ	density of convective fluid

Most of the engineering problems are governed by non-linear differential equations and in most cases it is difficult to solve them analytically. On the other hand, most of the previous analytic techniques face different problems in solving such non-linear equations.

Homotopy analysis method (HAM) was recently introduced by Liao [10]. It is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Many previous analytic methods have some restrictions in dealing with non-linear equations. For instance, in contrast to perturbation method, HAM is independent of any small or large parameters and or the existence of auxiliary parameter $\hbar$provides us with a simple way to control and adjust the convergence region which is a main lack of previous techniques. Also, HAM provides us with great freedom to choose different base functions to express solutions of a nonlinear problem [10].

As shown by Liao [10], HAM method logically contains some previous analytic techniques such as Adomian's decomposition method, Lyapunov's artificial small parameter method, and the δ-expansion method. Many authors [11–33] have successfully applied HAM in solving different types of nonlinear problems arising in heat transfer, fluid flow, oscillatory systems and etc.

2. Mathematical formulation

Consider the asymmetric buoyancy flow in a saturated porous medium below a cooled horizontal impermeable surface (Fig. 1) or above a heated surface (Fig. 2) with wall temperature being a function of radius.

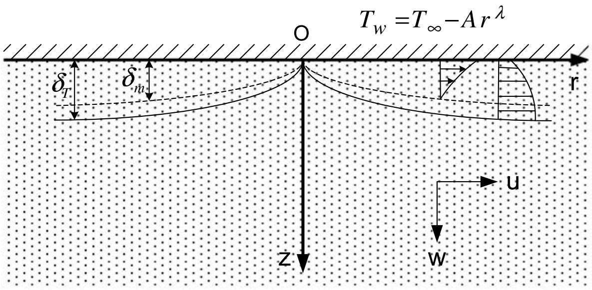

Fig. 1. Porous medium below a cooled horizontal impermeable surface.

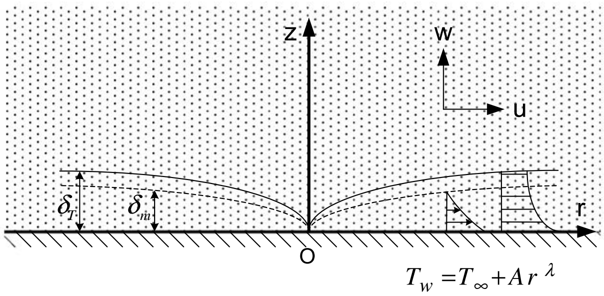

Fig. 2. Porous medium above a heated horizontal impermeable surface.

In Figs. 1 and 2, r and z are cylindrical coordinates in horizontal and vertical directions with positive *z-axis* pointing toward the porous medium. If we assume that (a) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium, (b) the temperature of the fluid is everywhere below boiling point, (c) properties of the fluid and the porous medium are constant and (d) the Boussinesq approximation is employed, the governing equations are given by

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial r}(rw) = 0, \tag{1}$$

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial r}, \tag{2}$$

$$w = -\frac{k}{\mu}\left(\frac{\partial p}{\partial z} \pm \rho g\right), \tag{3}$$

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)], \tag{4}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right), \tag{5}$$

where the "+" sign in Eq. (3) refers to the case of a heated impermeable surface facing upward while the "−" sign refers to the case of a cooled impermeable surface facing downward. In Eqs. (1)–(5), u and w are the velocity components in the horizontal and vertical directions respectively. The subscript "∞" refers to the condition at infinity. $\alpha = k/(\rho_\infty c)_f$ is the equivalent thermal diffusivity where the subscript "f" refers to convecting fluid.

The boundary conditions for the problem are

$$z = 0 \quad T_w = T_\infty \pm Ar^{\lambda} \quad w = 0, \tag{6}$$

$$z \to \infty \quad T = T_\infty \quad u = 0, \tag{7}$$

where constant $A > 0$ and the "+" and "−" signs in Eq. (6) are for a heated impermeable surface facing upward and for a cooled impermeable surface facing downward respectively. Eq. (6) shows that the prescribed wall temperature is a power function of radius from the origin.

The continuity equation is automatically satisfied by introducing the stream function ψ as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z} \quad \text{and} \quad w = -\frac{1}{r}\frac{\partial \psi}{\partial r}, \tag{8}$$

Eliminating $p$from Eqs. (2) and (3) by cross differentiation, the resulting equation in terms of ψ becomes:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 \psi}{\partial z^2} = \pm\frac{k\rho_\infty g\beta}{\mu}\frac{\partial T}{\partial r}. \tag{9}$$

Eq. (4), in terms of ψ, can be rewritten as

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + r\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\left[-\frac{\partial \psi}{\partial z}\frac{\partial T}{\partial r} + \frac{\partial \psi}{\partial r}\frac{\partial T}{\partial z}\right]. \tag{10}$$

The appropriate boundary conditions for Eqs. (9) and (10) are

$$z = 0 \quad T_w = T_\infty + Ar^{\lambda} \quad \frac{\partial \psi}{\partial r} = 0, \tag{11}$$

$$z \to \infty \quad T = T_\infty \quad \frac{\partial \psi}{\partial z} = 0. \tag{12}$$

Here we present the procedure for finding the similarity solution of the governing equations.

As was mentioned in the previous section, the foregoing equations are derived by using boundary layer approximation which is valid if: (a) $\frac{\partial}{\partial z} \gg \frac{\partial}{\partial r}$ and (b) $w \ll u$.

Under the assumption of boundary layer behavior, Eqs. (9) and (10) can be approximated by

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2} = \pm\frac{k\rho_\infty g\beta}{\mu}\frac{\partial T}{\partial r}, \tag{13}$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\left[-\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial T}{\partial z}\right]. \tag{14}$$

To seek similarity solutions of Eqs. (13) and (14) with boundary conditions (11) and (12), we now introduce the following dimensionless variables:

$$\eta = \left[\frac{k\rho_\infty g\beta A}{\mu\alpha}\right]^{1/3} z r^{(\lambda-2)/3} = (Ra_r)^{1/3}\frac{z}{r}, \tag{15}$$

$$\psi = \alpha\left[\frac{k\rho_\infty g\beta A}{\mu\alpha}\right]^{1/3} r^{(4+\lambda)/3} f(\eta) = \alpha(Ra_r)^{1/3} r f(\eta), \tag{16}$$

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty). \tag{17}$$

The governing Eqs. (13) and (14) and their appropriate boundary conditions are

$$f'' + \lambda\theta + \frac{(\lambda-2)}{3}\eta\theta' = 0, \tag{18}$$

$$\theta'' - \lambda\theta f' + \frac{(4+\lambda)}{3} f\theta' = 0, \tag{19}$$

$$\theta(0) = 1 \quad f(0) = 0, \tag{20}$$

$$\theta(\infty) = 0 \quad f'(\infty) = 0. \tag{21}$$

The acceptable range of λ to have a physically realistic problem is $1/2 \leqslant \lambda \leqslant 2$ [5].

The resulting coupled Eqs. (18) and (19) with boundary conditions (20) and (21) are solved through HAM and the results are compared with the numerical solution (NM).

3. Basic idea of HAM

In this section, we present the basic idea of homotopy analysis method which is used to solve the discussed coupled ordinary non-linear differential equation.

For brevity, here we consider only one non-linear equation in the following form:

$$\mathcal{N}[u(r,t)] = 0, \tag{22}$$

where $\mathcal{N}$ is a nonlinear operator, $u(r,t)$ is an unknown function and r and t are spatial and temporal independent variables, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in a similar way. By means of generalizing the traditional homotopy method, Liao [10] constructs the so-called zero order deformation equation

$$(1-p)\mathcal{L}[\phi(r,t;p) - u_0(r,t)] = p\hbar H(r,t)N[\phi(r,t;p)], \tag{23}$$

where $p \in [0,1]$ is the embedding parameter and, $\hbar \neq 0$ is a nonzero auxiliary parameter, $H(r,t) \neq 0$ is an auxiliary function, $\mathcal{L}$ is auxiliary linear operator, $u_0(r,t)$ is the initial guess which satisfies the initial or boundary conditions and $\phi(r,t;p)$ is an unknown function. It should be noted that there is a great freedom to choose auxiliary parameter$\hbar$, auxiliary function $H(r,t)$, initial guess $u_0(r,t)$ and auxiliary linear operator L. Beside such a great freedom, there are some fundamental rules which direct us to choose the mentioned parameters in a more efficient way. They are as follows: rule of solution expression, rule of coefficient ergodicity and the rule of solution existence [10].

Obviously we have the following relations:

$$\lim_{p\to 0}\phi(r,t;p) = u_0(r,t), \tag{24}$$

$$\lim_{p\to 1}\phi(r,t;p) = u(r,t)\,. \tag{25}$$

Thus, when p increases from 0 to 1 the solution $\phi(r,t;p)$ changes between the initial guess $u_0(r,t)$ and the solution $u(r,t)$. The Taylor series expansion of $\phi(r,t;p)$ with respect to p is

$$\phi(r,t;p) = u_0(r,t) + \sum_{m=1}^{+\infty} u_m(r,t)p^m, \tag{26}$$

where

$$u_m(r,t) = \frac{1}{m!}\frac{\partial^m \phi(r,t;p)}{\partial p^m}\bigg|_{p=0}. \tag{27}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, and the auxiliary function are so properly chosen, the series (26) converges at $p = 1$, one has

$$u(r,t) = u_0(r,t) + \sum_{m=1}^{+\infty} u_m(r,t), \tag{28}$$

which must be one of the solutions of original nonlinear equation, as proved by Liao [10].

According to the definition (27), the governing equation can be deduced from the zero-order deformation Eq. (23). Define the vector

$$\vec{u}_m = \{\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots, \vec{u}_n\}. \tag{29}$$

Differentiating Eq. (23) m times with respect to the embedding parameter $p$and setting $p = 0$ and finally dividing by $m!$, we will have the so-called mth order deformation equation:

$$\mathscr{L}[u_m(r,t) - \chi_m u_{m-1}(r,t)] = \hbar H(r,t) R(\vec{u}_{m-1}), \tag{30}$$

where

$$R_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1}\mathscr{N}[\phi(r,t;p)]}{\partial p^{m-1}}\bigg|_{p=0}, \tag{31}$$

and

$$\chi_m = \begin{cases} 0 & m \leqslant 1 \\ 1 & m > 1 \end{cases}, \tag{32}$$

so by applying inverse linear operator to both sides of the linear Eq. (30), we can easily solve the equation by means of symbolic computation software such as Mathematica, Maple, Matlab, and so on, and compute the generation constant by applying the initial/ boundary conditions.

4. Application

For the present problem, we choose the initial guesses of $f(\eta)$ and $\theta(\eta)$ in the following form, respectively:

$$f_0(\eta) = 0, \tag{33}$$

$$\theta_0(\eta) = \exp(-\eta). \tag{34}$$

We choose the following equations as the auxiliary linear operators:

$$\mathscr{L}_{\mathrm{f}} = \frac{\partial^2}{\partial \eta^2}, \tag{35}$$

$$\mathscr{L}_\theta = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}, \tag{36}$$

with the properties

$$\mathscr{L}_{\mathrm{f}}(C_1\eta + C_2) = 0, \tag{37}$$

$$\mathscr{L}_\theta(C_3 + C_4\exp(-\eta)) = 0, \tag{38}$$

where $c_i(i = 1,2,3,4)$ are coefficients.

According to Eqs. (18) and (19), we now define the non-linear operators as

$$\mathscr{N}_{\mathrm{f}}[F(\eta;p), \Theta(\eta;p)] = \frac{\partial^2 F(\eta;p)}{\partial \eta^2} + \lambda\Theta(\eta;p) + \frac{(\lambda-2)}{3}\eta\frac{\partial\Theta(\eta;p)}{\partial\eta} \tag{39}$$

$$\mathscr{N}_\theta[F(\eta;p), \Theta(\eta;p)] = \frac{\partial^2 \Theta(\eta;p)}{\partial \eta^2} + \frac{(\lambda+4)}{3}F\frac{\partial\Theta(\eta;p)}{\partial\eta} - \lambda\Theta(\eta;p)\frac{\partial F(\eta;p)}{\partial\eta} \tag{40}$$

where $p \in [0,1]$ is an embedding parameter and $F(\eta;p)$ and $\Theta(\eta;p)$ are real functions of η and p.

Let $\hbar_{\mathrm{f}}$ and $\hbar_\theta$ denote the nonzero auxiliary parameters, $H_{\mathrm{f}}(\eta)$ and $H_\theta(\eta)$ the nonzero auxiliary functions, respectively. We construct the zero-order deformation equations

$$(1-P)\mathscr{L}_{\mathrm{f}}[F(\eta;p)-f_0(\eta)] = p\hbar_{\mathrm{f}} H_{\mathrm{f}}(\eta)\mathscr{N}_{\mathrm{f}}[F(\eta;p),\Theta(\eta;p)], \tag{41}$$

$$(1-P)\mathscr{L}_{\theta}[\Theta(\eta;p)-\theta_0(\eta)] = p\hbar_{\theta} H_{\theta}(\eta)N_{\theta}[F(\eta;p),\Theta(\eta;p)]. \tag{42}$$

Subject to the following boundary conditions:

$$\Theta(0,p)=1 \quad F(0,p)=0, \tag{43}$$

$$\Theta(\infty,p)=0 \quad F'(0,p)=0, \tag{44}$$

where $p \in [0,1]$ is an embedding parameter. It is obvious that:

$$\Theta(\eta,0)=\theta_0 \quad F(\eta,0)=f_0 \quad \text{when } p=0, \tag{45}$$

$$\Theta(\eta,1)=\theta(\eta) \quad F(\eta,0)=f(\eta) \quad \text{when } p=1, \tag{46}$$

Thus as p increases from 0 to 1, $F(\eta,p)$ and $\Theta(\eta,p)$ deform from the initial guesses f_0, θ_0 to the solutions $f(\eta)$, $\theta(\eta)$ of Eqs. (18)–(21), respectively.

Assuming that the auxiliary parameters $\hbar_{\mathrm{f}}$, $\hbar_{\theta}$ and the auxiliary functions $H_{\mathrm{f}}(\eta)$, $H_{\theta}(\eta)$ are properly chosen so that the Taylor series of $F(\eta;p)$ and $\Theta(\eta;p)$ converge at $p = 1$, we then from Eqs. (45) and (46) have:

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta), \tag{47}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{+\infty} \theta_m(\eta), \tag{48}$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m F(\eta;p)}{\partial p^m}\bigg|_{p=0}, \tag{49}$$

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \Theta(\eta;p)}{\partial p^m}\bigg|_{p=0}. \tag{50}$$

Differentiating the zero-order deformation Eqs. (41) and (42) m times with respect to p then dividing by $m!$ and setting $p = 0$, we obtain the mth-order deformation equations as

$$\mathscr{L}_{\mathrm{f}}[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_{\mathrm{f}} H_{\mathrm{f}}(\eta) R_m^f(\eta), \tag{51}$$

$$\mathscr{L}_{\theta}[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_{\theta} H_{\theta}(\eta) R_m^{\theta}(\eta). \tag{52}$$

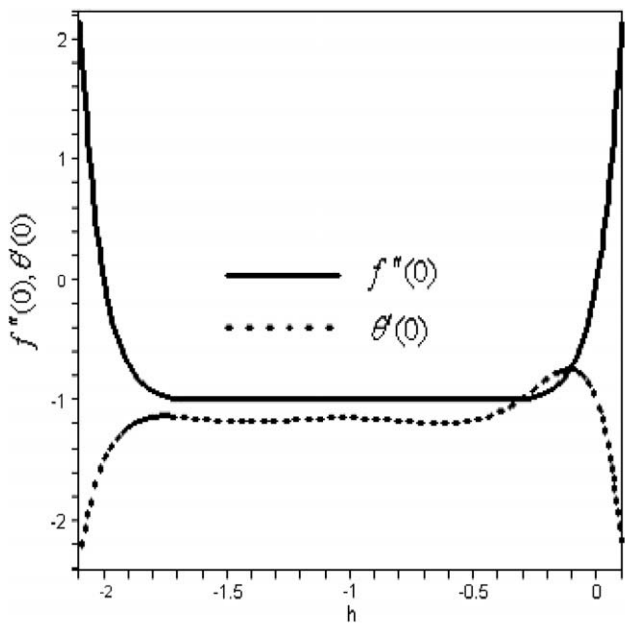

Fig. 3. The $\hbar$-curves of $f''(0)$ and $\theta'(0)$ at 12th order of approximation when $\lambda = 1$.

Subject to the boundary conditions

$$\Theta(0,p) = 0 \quad F(0,p) = 0, \tag{53}$$
$$\Theta(\infty,p) = 0 \quad F'(0,p) = 0. \tag{54}$$

For $m \geqslant 1$, where

$$R_m^f(\eta) = f''_{m-1} + \lambda\theta_{m-1} + \frac{(\lambda-2)}{3}\eta\theta'_{m-1}, \tag{55}$$

$$R_m^\theta(\eta) = \theta''_{m-1} + \frac{(\lambda+4)}{3}\sum_{k=0}^{m-1} f_{m-1-k}\theta'_k - \lambda\sum_{k=0}^{m-1} f'_{m-1-k}\theta_k, \tag{56}$$

and

$$\chi_m = \begin{cases} 0 & m \leqslant 1 \\ 1 & m > 1 \end{cases}. \tag{57}$$

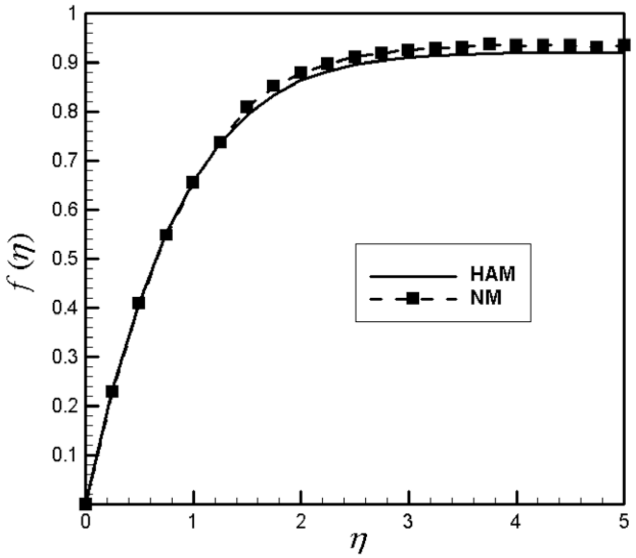

Fig. 4. Comparison between the HAM and numerical results of $f(\eta)$ for $\lambda = 1$ and $\hbar = -0.6$.

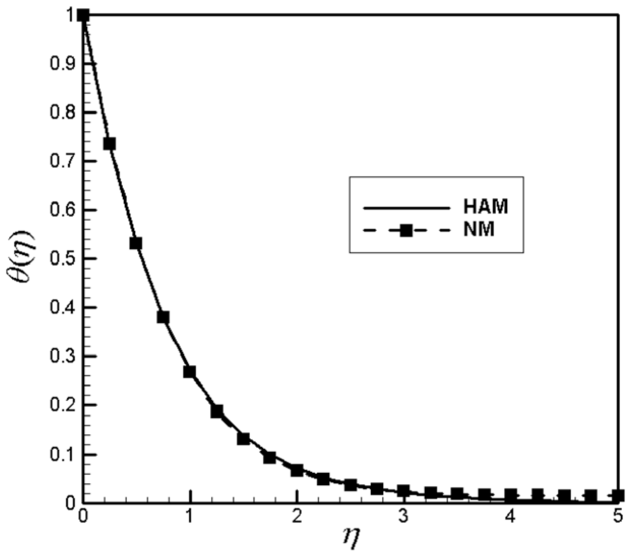

Fig. 5. Comparison between the HAM and numerical results of $\theta(\eta)$ for $\lambda = 1$ and $\hbar = -0.6$.

We have chosen $H_f = H_\theta = e^{-\eta}$ as the auxiliary functions in the present analysis. Then we have found the solution by using Maple symbolic computation device. Here, we present the first two expressions for $f(\eta)$ and $\theta(\eta)$. The subsequent terms in the series solution were too long to be mentioned here.

$$f_1(\eta) = \frac{1}{3}h_f\left(\frac{1}{4}e^{-2\eta}\eta + e^{-2\eta} - 1\right), \tag{58}$$

$$\theta_1(\eta) = \frac{1}{2}h_\theta\left(e^{-2\eta} - e^{-\eta}\right), \tag{59}$$

$$f_2(\eta) = -\frac{1}{648}h_f(-48h_f e^{-3\eta}\eta - 140e^{-3\eta}h_f + 27e^{-2\eta}h_f\eta + 108e^{-2\eta}h_f - 54e^{-2\eta}\eta - 216e^{-2\eta} + 32h_f + 216), \tag{60}$$

$$\theta_2(\eta) = \frac{1}{5184}h_\theta e^{-\eta}(1728e^{-2\eta}h_\theta + 144e^{-\eta}h_\theta + 12h_\theta e^{-3\eta}\eta + 19e^{-3\eta}h_\theta + 2592e^{-\eta} - 1891h_\theta - 2592). \tag{61}$$

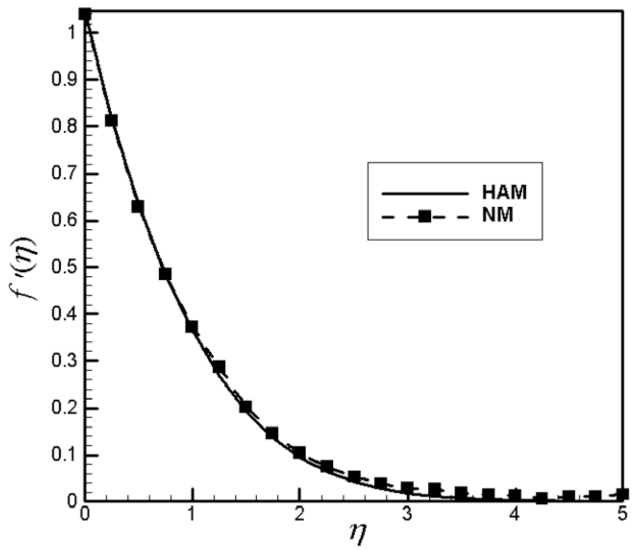

Fig. 6. Comparison between the HAM and numerical results of $f'(\eta)$ for $\lambda = 1$ and $\hbar = -0.6$.

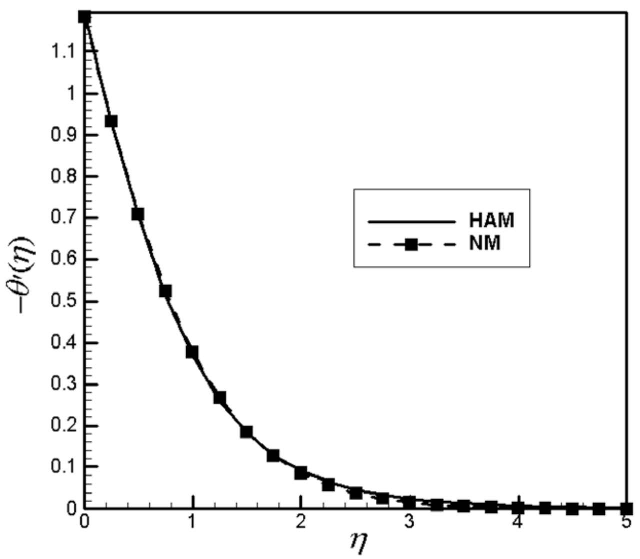

Fig. 7. Comparison between the HAM and numerical results of $-\theta'(\eta)$ for $\lambda = 1$ and $\hbar = -0.6$.

5. Results

Our solution series of $f(\eta)$ and $\theta(\eta)$ contain the auxiliary parameters h_f and h_θ, respectively. We should ensure that the mentioned series converge to the solution of the problem by properly choosing the relevant auxiliary parameters i.e. h_f and h_θ, because as pointed out by Liao [10], the convergence and rate of approximation for the HAM solution strongly depends on the values of auxiliary parameters. In doing so, we utilize the so-called $\hbar$-curves and find the appropriate ranges for h_f and h_θ. We plot $f''(0)$ and $\theta'(0)$ for various values of $\hbar$ for case of $\lambda = 1$. For simplicity, we let the two auxiliary parameters to be equal i.e. $h = h_f = h_\theta$ (see Fig. 3).

From Fig. 3, it is clear that the convergent result can be obtained when $-1.6 \leqslant \hbar \leqslant -0.5$. So, we can choose appropriate value for $\hbar$in this range.

For $\hbar = -0.6$ and $\hbar = -0.8$, we plotted the results and compared them with the numerical solutions for $\lambda = 1$ and $\lambda = 1.5$, respectively. It should be mentioned that HAM solutions are valid for all acceptable values of λ i.e. $1/2 \leqslant \lambda \leqslant 2$ [5], but here we just present the solution for two values of λ (see Figs. 4–11).

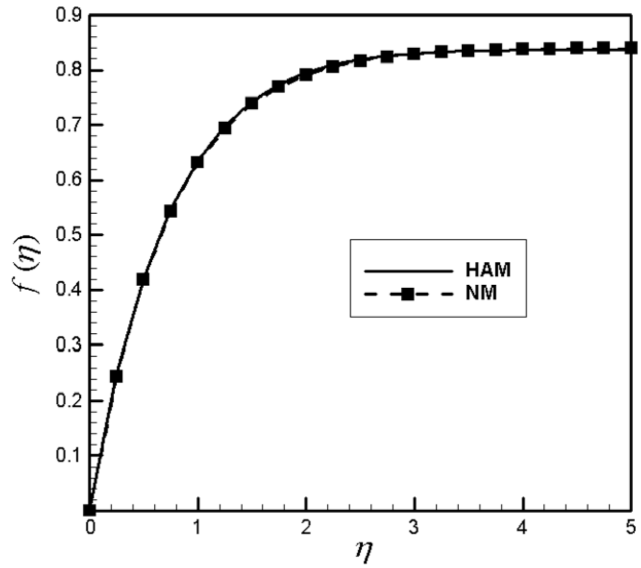

Fig. 8. Comparison between the HAM and numerical results of $f(\eta)$ for $\lambda = 1.5$ and $\hbar = -0.8$.

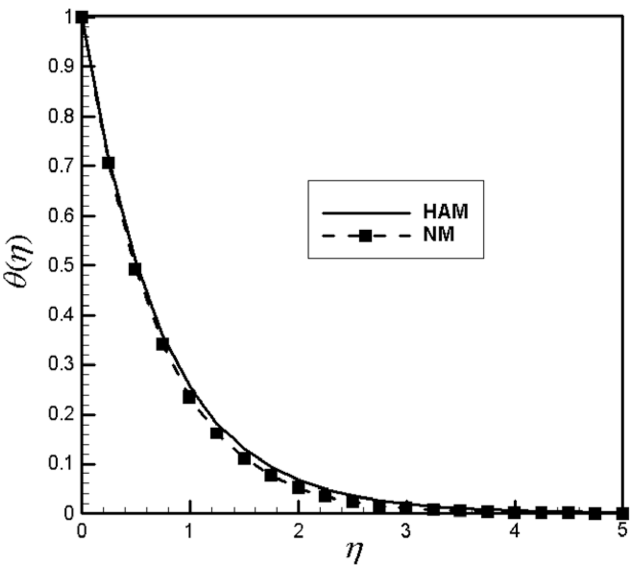

Fig. 9. Comparison between the HAM and numerical results of $\theta(\eta)$ for $\lambda = 1.5$ and $\hbar = -0.8$.

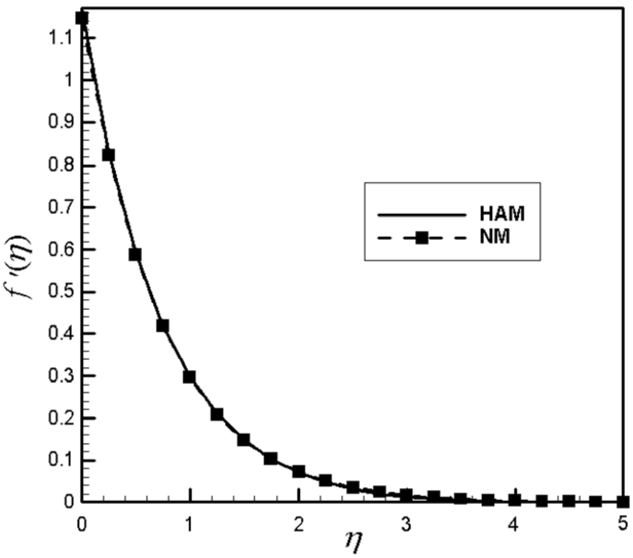

Fig. 10. Comparison between the HAM and numerical results of $f'(\eta)$ for $\lambda = 1.5$ and $\hbar = -0.8$.

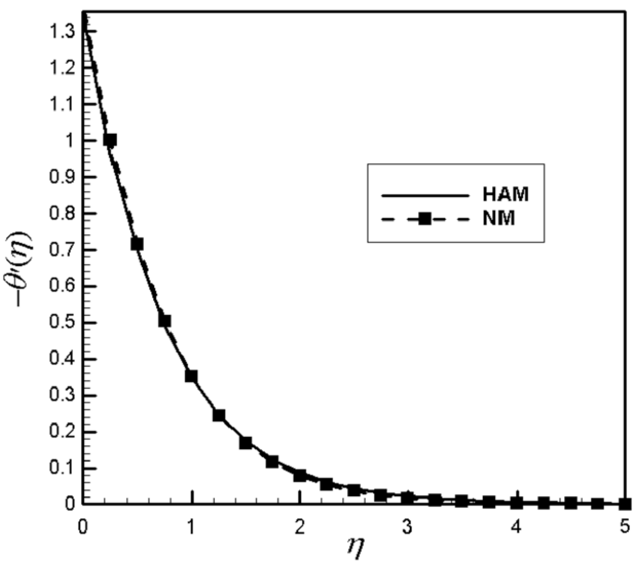

Fig. 11. Comparison between the HAM and numerical results of $-\theta'(\eta)$ for $\lambda = 1.5$ and $\hbar = -0.8$.

6. Conclusion

In this study, free convection over a horizontal impermeable flat plate embedded in a saturated porous medium was analyzed using HAM. This method provides highly accurate analytic solutions of nonlinear problems compared with previous analytic and numerical methods. It has been attempted to show the role of $\hbar$ as an important parameter in determining and augmenting the convergence of the solution and also capabilities and wide-range applications of the homotopy analysis method. The results are shown graphically for two values of temperature exponent parameter λ and they are compared with the numerical solutions. The comparison shows great agreement between the results.

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