Attempts to identify distant underwater explosions by their acoustic signals based on spherical bubble models may produce erroneous results in cases where the explosion occurs in shallow water, or near a rigid surface. An experimental and numerical study of the effects of bubble distortions, due to gravity or proximity to a surface, on the bubble’s acoustic signature is presented. Measurements from high-speed visualizations and acoustic signals are presented which show that the effect of bubble distortion near a rigid surface is to increase the first period, weaken the first bubble pulse, and affect significantly the second period.

INTRODUCTION

Underwater explosions produce bubbles of gas that alternately expand and collapse in a series of pulsations, generating a train of acoustic pulses, with the initial shock wave plus a few additional “bubble pulses” being typically detectable. These pulses are characteristic of underwater explosions, serve to distinguish them from other sources of underwater sound, and can be used to help identify certain parameters of the explosion.

In the absence of gravity effects or nearby objects, the bubbles remain spherical, and their volume oscillations can be predicted analytically. Rayleigh [1] analyzed the collapse of empty spherical cavities and found the time of collapse to be

\[ T_{\text{collapse}} = 0.915 R_{\text{max}} \sqrt{\frac{\rho}{P_\infty}}, \]  

where \( R_{\text{max}} \) is the initial radius of the cavity, and \( \rho \) and \( P_\infty \) are the density and pressure of the ambient fluid. The effects of vapor pressure, noncondensible gas, surface tension, viscosity, and a time-varying ambient pressure can be included, to produce the Rayleigh-Plesset equation [2], the equation of wall motion for a collapsing and rebounding spherical bubble. For given conditions, the period of the bubble’s oscillation can be predicted from the equation. Using the unsteady Bernoulli equation, one can also predict pressures around the bubble.

If the explosion occurs near an object or at shallow depths where the effect of gravity is strong, the bubble collapses asymmetrically, resulting in reentrant jet and toroidal bubble formation, or if both effects are strong and in opposite directions, a pear shape and bubble splitting [3, 5], and the bubble does not produce the same series of pulses as it would in deep submergence and away from boundaries. Thus, attempts to characterize distant underwater explosions by their acoustic signals based on the spherical bubble assumption may produce erroneous results.
Results of an experimental and numerical study of the effects of gravity and of a nearby surface on bubble acoustic signals are presented. The effect of a nearby flat rigid surface is quantified in terms of the normalized standoff distance,

$$X = \frac{x}{R_{\text{max}}}$$

where $x$ is the standoff distance, i.e. the shortest distance from the explosion center to the surface, and $R_{\text{max}}$ is the radius the bubble would achieve if the explosive were detonated in an infinite medium at the same depth. For large values of the standoff parameter, the bubble is essentially spherical and unaffected by the boundary. As the standoff distance is reduced, the bubble becomes more and more affected by the boundary. At values near unity, the bubble nearly touches the surface on its first expansion, and at values near zero, the first bubble pulsation is hemispherical. Examples of this behavior for small-scale explosions generated by a spark in a vacuum cell [4] are shown in Figure 1.

The effect of gravity is quantified in terms of the Froude number [5],

$$\mathcal{F} = \frac{P_\infty - P_v}{2 \rho g R_{\text{max}}}.$$  

This parameter is the ratio of the difference between the ambient pressure and the vapor pressure to the difference in pressure between the top and bottom of the bubble. For large values of Froude number, the effect of gravity is relatively weak, and the bubble remains spherical. For small Froude numbers, bubble distortion due to gravity is important.

Figure 1. Spark-generated bubbles over a rigid surface. Three different times are shown—the time of the first volume maximum, just prior to the first collapse, and the time of the second volume maximum—for three different standoff distances, with standoff distance increasing from bottom to top. The ambient pressure was approximately 60 mbar, and $R_{\text{max}}$ was approximately 15mm.
EXPERIMENTAL APPROACH

Bubbles generated by the underwater discharge of a high-voltage charge between two coaxial electrodes contained within a transparent cell were used as laboratory-scale models of underwater explosions [4]. By reducing the pressure in the cell, relatively large and slow bubbles were generated, and the transparent walls allowed recording of the bubble dynamics by a high-speed digital camera (Redlake Imaging model PCI8000S). The acoustic signals of the bubbles were measured with a quartz transducer having a 1μs rise time (PCB Piezotronics model 102A03).

The energy of the bubble was controlled by adjusting the voltage supplied to the capacitor of the spark generator. Variations in the discharge output were removed from the data by measuring $R_{max}$ and normalizing periods using a characteristic time (equal to the Rayleigh collapse time omitting the factor 0.915) as in Eq. (4). The bubble’s maximum radius was also necessary to normalize the standoff distance, and it was obtained by direct measurement from the video corresponding to each signal. In cases where the bubble was significantly distant from the bottom, it retained a spherical shape through the time of its maximum volume and a radius could be easily measured. In cases where the bubble was distorted by the presence of the plate, an equivalent radius was used, which was obtained for each case by assuming the bubble to be axisymmetric, and calculating its volume as the volume of a solid of revolution of the observed outline around its vertical axis. The ambient pressure was calculated based on the atmospheric pressure in the vacuum cell and the depth of the electrodes, and the vapor pressure was calculated based on the ambient water temperature. With the quantities $R_{max}$, $P_{amb}$, and $P_v$, all times, $t$, and distances, $r$, were normalized using

$$\bar{t} \equiv \frac{t}{R_{max} \sqrt{\frac{\rho}{T_{max}}} - P_{v}}$$

$$\bar{r} \equiv \frac{r}{R_{max}}$$

NUMERICAL APPROACH

DYNAFLOW’s commercial code 2DYNAPS® [6] was used to conduct simulations of bubbles under conditions matching those of the spark tests. 2DYNAPS® is based on the boundary element method for solving potential flows with arbitrary boundaries, and calculates the motion of one or more interfaces in an axisymmetric geometry. Each interface is discretized in a meridional plane, each node is advanced at each time step using the velocities and the unsteady Bernoulli equation, and the new velocity potentials and normal velocities are calculated using a discretized version of Green’s equation [6]. In cases where a re-entrant jet impacts the opposite wall of the bubble, the dynamic cut relocation algorithm of Best [7] is used to continue computations with a new toroidal bubble. For laboratory-scale simulations, the initial conditions of the bubble were specified as the same ambient pressure as the corresponding experimental test, and an initially spherical explosion bubble with a radius and pressure such as to produce an $R_{max}$ value of 1.3 cm, which is a typical value for the spark tests in the pressure range tested.

RESULTS

Figure 2 shows an example series of pressure signals from bubbles generated by spark discharges at varying distances above a plate, with each signal offset vertically in the figure by an amount proportional to its corresponding standoff distance. The time has been normalized as in Eq. (4) in order to compensate for variations in the spark discharge energy. Each signal contains initial strong fluctuations composed of a combination of the initial shock wave and electrical noise associated with the spark discharge, followed by two and sometimes three detectable peaks generated by the bubble collapses. Two principal differences among the signals can be observed. First, the timing of the pulses appears in general delayed as the bubble approaches the plate. Second, the first bubble pulse decreases in strength dramatically as the bubble approaches the plate.
Figure 2. Pressure signals from a spark generated bubble with varying standoff distance. Signals were measured using an ambient pressure of one atmosphere and a maximum bubble radius of about 6 mm, and a quartz pressure transducer several inches to one side. All signals have been offset vertically by an amount proportional to the respective standoff distance.

The period of an explosion bubble is given classically as a function of explosive amount and ambient pressure by a formula of the form

$$T = KW^{1/3}Z^{-5/6}.$$  \hspace{1cm} (6)

In this expression, \(W\) is the mass of the explosive, \(Z\) is the hydrostatic head at the center of the explosion, and \(K\) is a parameter of the particular explosive [8]. This formula, however, assumes a spherical bubble far removed from the effects of boundaries. For bubbles at moderate distances from a rigid surface, Chahine and Bovis [9] have demonstrated using asymptotic expansion that

$$T^* \approx 0.915 \left(1 + \frac{1}{4X} \int_0^{\nu(t)R} R(t) dt \right) = 0.915 \left(1 + \frac{0.79}{4X} \right),$$  \hspace{1cm} (7)

where \(T^*\) is the normalized period of the bubble in the presence of the surface, and \(X\) is the normalized standoff distance. For small values of standoff distance, however, this first-order approximation is not valid.

In order to investigate the effects of proximity to a rigid boundary over the entire range of standoff distances, several series of spark tests were conducted using approximately constant energy, but varying standoff distances, including a concentrated series of measurements for values of \(X\) less than one, and these results were compared to the results from a series of simulations using 2D DYNAFS\textsuperscript{\textregistered}. The measured and calculated first bubble periods are shown in Figure 3, along with a curve calculated from equation (7).
Both the measurements and the simulations agree with the prediction of first-order asymptotic expansion for values of $X$ larger than one, where the period is slightly greater than twice the characteristic time, as expected. As the bubble nears the bottom, the period shows a steady gradual increase, up to a value at the bottom approximately 20-25% greater than its “free” value. This increase is consistent theoretically with the additional energy due to an image bubble on the opposite side of the surface. If a hemispherical bubble is generated exactly at the surface, it will act with its mirror image as a single spherical bubble having twice the energy as the actual bubble, and according to the period-energy relation will have a period greater than the actual bubble if it were spherical by a factor of $2^{1/3}=1.26$.

Figure 3. First bubble period as a function of standoff distance. Experimental data was collected for several cell pressures, and simulations were conducted using an ambient pressure of 50 mbar plus a correction for depth, and an $R_{max}$ value of 1.3 cm.

Figure 4. Normalized second period as a function of standoff. Experimental data is shown from the same set of tests as in Figure 3, and has been normalized using Eq. (4) and Eq. (5).
The period of the second cycle, shown in Figure 4, depends on the history of the bubble through the first cycle and exhibits more variation with standoff than does the first period. The second period has been normalized using the same characteristic time as the first period in each case. At large standoff distances, the period is nearly half that of the first cycle, which according to the period-energy relation for spherical bubbles corresponds to an energy of \((\frac{1}{2})^3\) or 12.5% that of the original energy, or an energy loss at the first collapse of about 88%. This corresponds to typical energy losses at the first collapse for spherical underwater explosion bubbles at shallow depths [10], and is consistent with reported values for laser-generated cavitation bubbles [11]. As the standoff distance decreases to a value of \(X=1\), the second period increases to a large fraction of its original value, suggesting a reduction in the bubble energy loss during the first collapse. A reduction in energy loss is also suggested by a reduction in emitted acoustic energy, as is indicated in Figure 5, which shows a pronounced minimum in the peak pulse pressure near \(X=1\). However, a change in period is also to be expected simply due to the formation of a vortex ring bubble, whose period is known to be larger than that of a spherical bubble of equal energy [12]. As the standoff distance decreases from \(X=1\) to \(X=0\), the period again decreases. This behavior of exhibiting an extremum near \(X=1\) is also shown in the measurements of Lindau and Lauterborn [13], who measured maximum and minimum volumes of laser-generated bubbles near a plate and found that the ratio of maximum to minimum volume near \(X=1\) was reduced by roughly two orders of magnitude from its value very near and very far from the plate. This extremum behavior is also displayed in the value of peak pressure during the first bubble pulse shown in Figure 5.

Note that the value of peak pressure shown in Figure 5 is probably lower than the actual value due to the extreme brevity of the pulse, and the relatively large size and slow response of the quartz transducer. The pressures in all cases were measured several inches to the side, and were normalized to correspond to a three inch separation between electrodes and transducer, using an assumed \(1/R\) dependence of pressure with distance.

Since standoff values much less and much greater than the bubble size correspond approximately to hemispherical and spherical symmetry, while standoff values nearly equal to the bubble size result in highly non-spherical bubbles and development of strong linear and rotational motion, it seems reasonable to explain the extrema in all of these properties in terms of bubble distortion during the first cycle and energy loss during the first collapse. For hemispherically and spherically symmetric bubble collapse, all of the energy of the fluid is directed into compression of the gaseous contents of the bubble, producing small volumes and large pressures, and large energy losses due to acoustic radiation. For values of \(X=1\), much of the fluid energy goes into formation of a jet and toroidal bubble, and the asymmetric collapse deprives the gas of some of its compression energy, producing larger minimum volumes, smaller pressures, and weaker acoustic emission.

![Figure 5](image_url)

**Figure 5.** First bubble pulse strength as a function of standoff. The maximum pressure recorded by the transducer during the first bubble pulse is shown from the same tests as in Figure 3 and Figure 4.
Larger second-cycle bubble volumes as well as larger second periods are also observed near \( X=1 \). Figure 6 shows approximate measurements of the bubble’s equivalent radius at the second volume maximum \((R_{max2})\) for a number of the experimental tests, as a fraction of the first \( R_{max} \) value. Similar to the second period, the second maximum radius displays a maximum near \( X=1 \). Furthermore, the similarity between the radius ratio and the ratio of second to first period, shown in Figure 7, suggests that the period scales approximately as the equivalent maximum radius for nonspherical bubbles as well as spherical, and both are decreased in nearly the same proportion by the loss of energy during the first collapse.

![Figure 6. Ratio of second maximum radius to first maximum radius as a function of standoff distance. Data is shown from the same set as shown in previous figures.](image1)

![Figure 7. Ratio of second period to first period as a function of standoff distance. Data is shown from the same experimental tests as shown in previous figures.](image2)

One possible explanation for the minimum in the second bubble period near normalized standoff values of approximately 0.3 is increased energy loss due to broad jet impact. The impact process is too rapid to be captured by the digital camera, but the acoustic signal at small standoff distances typically shows a pressure peak preceding the minimum volume peak. The precursor peak is known to occur as a result of jet impact [15, 16]. Figure 8 shows typical peak profiles for normalized standoff distances of 0.06, 0.30, 0.45, and 0.69. The initial peak is evident for \( X=0.06 \) and quite pronounced at \( X=0.3 \). Also evident is the decrease in peak amplitude for standoff distances near \( X=1 \). A very unsteady peak such as Figure 8(c) is often observed for standoff values in the neighborhood of \( X=1 \), and sometimes for bubbles strongly influenced by gravity as well, and is probably a result of partial or complete bubble breakup during collapse. Bubble collapse is an unstable process, and any perturbations, such as disruption of flow around an electrode, or any other object present in a non-ideal flow field, will promote breakup of the bubble. This breakup has sometimes been observed in overhead views to produce daughter bubble fragments which collapse independently, sometimes producing several distinct collapse peaks. At a standoff distance of \( X=0.69 \), the jet impact peak becomes predominant.

**Effect of Gravity**

The effect of gravity on explosion bubbles is similar to that of a nearby surface, in that it causes bubble distortion, formation of a jet and then toroidal bubble formation, and bubble migration. Some preliminary results showing the effect this gravity-induced distortion has on bubble period are presented in Figure 9. The range of Froude numbers corresponds approximately both to the range producible in the spark-generated bubble cell, and to typical full-scale underwater explosions.

At large Froude numbers, the bubbles are nearly spherical and the period, normalized for bubble size and ambient pressure using the characteristic time as in Eq. (4), should be nearly constant and approximately equal to two. The experimental data, the simulations using 2DYNASFS\textsuperscript{®}, and the one-dimensional spherical model given by the
Rayleigh-Plesset equation all agree approximately with this value, although the simulations show a distinct increasing trend with Froude number. Since this appears in the one-dimensional model as well, the cause is evidently not bubble distortion but is due to the gas pressure, i.e. the ratio between bubble gas pressure and ambient pressure. The Froude number was varied in the simulations as it was in the experiments, by varying the ambient pressure, and this affects both bubble size and the characteristic time, which are used in the normalization. At low Froude numbers, simulations using 2DYNAFS predict that the period will rise as the distorting influence of gravity becomes more rapid, as it does for the distorting influence of a nearby rigid surface. The reason for the systematic discrepancy between the data and the numerical predictions at low Froude numbers is possibly connected to an increasing influence of the free surface, which is known to shorten the bubble period, on the experimental bubbles at low Froude number.

Like bubbles near a rigid surface, the second period of bubbles in gravity, shown in Figure 10, displays a dependence on bubble distortion, although the variation is weaker over the parameter range. At large Froude numbers, the second period is nearly half that of the first period, indicating approximately an 88% energy loss, as for bubbles distant from a rigid surface. As the Froude number decreases and bubble distortion becomes more rapid, the second period begins to increase slightly, suggesting reduced energy loss during the first collapse. Reduced energy loss is also suggested by reduced peak pressures at the first collapse, shown in Figure 11.

Unlike bubbles near a rigid surface, bubble shape and behavior during the second cycle is roughly similar to that during the first cycle. Upon the first rebound, the bubble typically resumes an approximately spherical shape, and the second collapse proceeds in a similar manner to the first, although with a reduced size and energy. The relative unimportance of geometry for these cases is also shown in the second period, renormalized using a characteristic time based on the second maximum radius rather than the first. This renormalized second period displays a nearly constant value nearly equal to two, in agreement with the spherical bubble model.
Figure 9. First bubble period as a function of Froude number. Measurements of spark-generated bubbles under various pressures are compared to simulations using 2DyNaFS© and to the prediction of the Rayleigh-Plesset Equation for the same conditions.

Figure 10. Second bubble period as a function of Froude number. The renormalized period was normalized using a characteristic time based on the bubble’s second maximum radius. The data is from the same experimental tests as in Figure 9.

Figure 11. First bubble pulse pressure versus Froude number. The maximum pressure recorded by the pressure transducer during the first bubble pulse is shown for the same tests as in Figure 9.
SUMMARY

We have presented measurements of the acoustic signals of laboratory-scale underwater explosion bubbles, as well as simulations for similar conditions, which demonstrate the effects of bubble distortions on the acoustic signals. The influence of a nearby rigid surface increases the first period, up to a value for hemispherical bubbles on the surface approximately 25% greater than the free field value, and increases the second period significantly near a normalized standoff value of $X = 1$, due to jet and toroidal bubble formation during the first collapse. This increase in the second period corresponds to reductions in the first bubble pulse pressure and increases in the second cycle bubble size near $X = 1$. Distortions due to gravity cause similar, though much less pronounced, alterations to bubble pulse periods and pressures.

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