Dynamics of Dispersed Bubbly Flow over a Lifting Surface: Gas Diffusion and Bubble Breakup Effects
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ABSTRACT
The modification of the bubble nuclei population by the presence of a finite-span hydrofoil is modeled numerically using an Eulerian-Lagrangian approach. The unsteady liquid flow field is simulated using Navier Stokes equations, while the bubbly flow initiating from nuclei in the free stream and emitted from boundaries is tracked using a Lagrangian approach. The effects of including gas diffusion and bubble breakup on nuclei distribution downstream of hydrofoil are studied. The inclusion of gas diffusion is found to significantly increase the size of the bubble downstream. Inclusion of boundary nucleation is found to significantly increase the total number of large bubbles collected in the wake as compared to the free nuclei alone. The study also compares the results obtained from two different bubble breakup models, one is based on instability analysis and the other one based on experimental observations. Both breakup models are shown to result in a very significant increase in the number of small and mid-size bubbles and a reduction in the number of bubbles of large size.

INTRODUCTION
The generation and entrainment of bubbles from a surface ship is of interest to marine applications. Besides the commonly known air entrainment sources due to free surface and hull interactions, such as breaking and spilling waves at the bow and stern and boundary layer entrainment along the hull, another potential source is bubble production by highly loaded devices such as propellers or other appendages with lifting surfaces. Photographic evidence has shown that a relatively large amount of bubbles is generated downstream of a lifting surface even in absence of visible upstream bubbles. The presence of bubbles can be attributed to the fundamental observation that any water contains microscopic sub-visual bubble nuclei that act as seeds for many phenomena in mechanical and chemical engineering such as: cavitation, boiling, gas transfer, chemical reactions, etc. These nuclei can be very small microscopic bubbles (Franklin 1992), particles with gas filled crevices, or entrapped gaseous micro- or nano-bubbles at the rigid boundaries (Mørch 2009, Yount et al. 1984). On encountering low pressure conditions these nuclei grow and oscillate causing cumulative non-condensable gas transfer into the bubbles. Therefore, it is essential that the analysis of bubble distributions and dynamics needs to consider both nuclei sources, i.e. suspended nuclei in water and boundary nucleation.

With the propeller flow field obtained from a RANS solver, Hsiao & Chahine (2012) developed a Discrete Singularity Model (DSM) to track all individual bubbles while considering the actual bubble volume variation due to pressure balance at the bubble wall along its trajectories. A gas diffusion model solving the gas transport equation based on thin boundary layer assumption (Plesset & Zwick 1952) was incorporated with the DSM to study the effects of non-condensable gas transfer on cavitation bubble dynamics. This study explained the observation of a bubbly field behind a propeller with no visible presence of bubbles upstream of the propeller and concluded that explosive bubble growth and violent collapse is an essential ‘catalyst’ to enable significant net influx of gas into the bubble originally dissolved in the liquid.

A more recent study (Hsiao et al. 2017a) has extended the previous study to an unsteady finite-span hydrofoil flow. This study employed a Eulerian-Lagrangian approach, solved the unsteady flow in a Eulerian frame and provided the flow field at each time step to the Lagrangian solver for tracking all the discrete bubbles. To enable tracking a very large number of bubbles simultaneously, this study used a finite difference scheme to solve the gas transport equation instead of using the boundary integral solution, which requires integration over the whole history of the bubble dynamics to compute the amount of gas inside the bubble. This study, not only revealed the importance of accounting for the instantaneous
flow field on the dispersed bubbly flow, but also demonstrated the effect of bubble breakup on the bubble size distribution downstream of the hydrofoil by including a heuristic breakup model in the absence of theoretical or empirical basis for the parameters.

The current study is to further investigate the effects of bubble breakup on the bubble size distribution by adopting and/or developing a better bubble breakup model. Our attempts include an analytical model based on stability analysis of the bubble shape and a statistical model based on experimental observations.

**NUMERICAL APPROACH**

The Eulerian-Lagrangian two-phase flow framework employed in this study has been extensively applied and documented in our previous studies. These include modeling of tip vortex cavitation inception on a propeller (Hsiao & Chahine 2004, 2008), investigations of the effects of a propeller flow on bubble size distribution in water (Hsiao & Chahine 2012), bubble entrainment in plunging jets (Hsiao et al. 2013), wave propagation in bubbly media and bubble cloud collapse studies (Ma et al. 2018, 2015a,b; Raju et al. 2011), and modeling sheet to cloud cavitation (Hsiao et al. 2017b) etc. The procedure is based on the following basic statements:

1. The dynamics and motion of the individual bubbles in the flow field are controlled by the two-phase medium local properties and gradients.
2. These local properties of the mixture (void fractions and local densities) are determined by the instantaneous bubble size and position distribution.
3. The mixture flow field has an evolving density distribution, which is space and time dependent, and satisfies mass and momentum conservation.

This two-phase flow framework allows both two-way and one-way coupling between the continuum-based model and the discrete bubbles model.

**Eulerian Continuum Mixture Model**

The two-phase flow continuum is solved using the Navier-Stokes equations solver, 3DYNAFS-VIS©, which satisfies the continuity and momentum equations:

\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) = 0, \tag{1}
\]

\[
\rho_m \frac{D \mathbf{u}}{D t} = -\nabla p + \mu_m \nabla^2 \mathbf{u}, \tag{2}
\]

where the subscript \(m\) represents the mixture properties. \(\mathbf{u}\) is the mixture velocity and \(p\) is the pressure. The mixture density, \(\rho_m\), and the mixture viscosity, \(\mu_m\), can be expressed as functions of the gas volume fraction, \(\alpha\), through:

\[
\rho_m = (1 - \alpha) \rho_l + \alpha \rho_g, \quad \mu_m = (1 - \alpha) \mu_l + \alpha \mu_g, \tag{3}
\]

where the subscript \(l\) represents the liquid and the subscript \(g\) represents the gas. The two-phase medium density and viscosity are time- and space-dependent.

The system of equations is solved by an artificial compressibility method (Chorin 1967) in which a derivative of the pressure using the pseudo-time, \(\tau\), multiplied by the inverse of the artificial compressibility factor, \(\beta\), is added to the continuity equation as:

\[
\frac{1}{\beta} \frac{\partial p}{\partial \tau} + \frac{\partial \rho_m}{\partial \tau} + \nabla \cdot (\rho_m \mathbf{u}) = 0. \tag{4}
\]

As a consequence, a hyperbolic system of equations is formed and can be solved using a time marching scheme. The solution is iterated in the pseudo-time until convergence. To obtain a time-dependent solution, a Newton iterative procedure is performed at each physical time step in order to satisfy the continuity equation.

**Lagrangian Discrete Bubble Model**

The Lagrangian discrete bubble model, 3DYNAFS-DSM©, uses singularities to model the bubbles (Hsiao et al. 2003). This model has been shown to produce accurate results when compared to full 3D two-way interaction computations (Hsiao & Chahine 2004). The source term, representing bubble volume oscillations, uses a Surface Averaged Pressure (SAP) modified version of the Keller-Herring equation (Keller & Kolodner, 1956) to describe the bubble dynamics:

\[
\left(1 - \frac{1}{c_m^2}ight) \frac{d}{dt}\left(\frac{R^2}{2c_m^2}\right) + \frac{1}{\rho_m}\left[1 + \frac{c_m}{c_v} \frac{d}{dt}\left(\frac{R}{c_v}\right)\right] \frac{R}{dt} - \frac{2 \gamma}{R} - \frac{4 \mu_m}{R}, \tag{5}
\]

where \(c_m\) is the local sound speed in the mixture. \(R\) and \(R_0\) are the bubble radii at times \(t\) and 0, \(\rho\) is the liquid vapor pressure, and \(u_{enc}\) and \(p_{enc}\) are respectively the averages of the liquid velocities and pressures over the bubble surface. The slip velocity, \(u_s\), is the difference between \(u_{enc}\) and the bubble translation velocity, \(\mathbf{u}_b\), i.e. \(\mathbf{u}_s = u_{enc} - u_b\). The gas pressure, \(p_g\), is obtained from a polytropic compression law if no gas mass transfer effects are taken into account. When gas diffusion is included, the gas pressure is obtained from the solution of the gas pressure.
diffusion problem and energy balance, described below.

The bubble trajectory is obtained from the following bubble motion equation:

\[ \frac{du_b}{dt} = \left( \frac{\rho_b}{\rho_s} \right) \left[ \frac{3}{8R} C_D |u_b| u_b + \frac{1}{2} \left( \frac{du_m}{dt} - \frac{d\Omega}{dt} \right) + \frac{3R}{2} \frac{\nabla p}{\rho_b} + \frac{(\rho_b - \rho_l)}{\rho_l} g + \frac{3C_L}{4\pi R} \frac{\nabla \times u_b \times \Omega}{\sqrt{|\Omega|}} \right] \tag{6} \]

where \( C_D \) is the drag coefficient, given by an empirical equation such as from (Haberman & Morton 1953). \( C_L \) is the lift coefficient and \( \Omega \) is the vorticity vector. The first right hand side term is a drag force. The second and third terms account for the added mass. The fourth term accounts for the presence of a pressure gradient, while the fifth term accounts for gravity and the sixth term is a lift force (Saffman 1965).

**DISPERSED BUBBLY FLOW SIMULATIONS**

**Unsteady Single Phase Flow**

We consider the unsteady flow field over a NACA0015 finite-span rectangular hydrofoil with a round tip and an aspect ratio based on semi-span of 2. The selected computational domain has all far-field boundaries located six chord lengths away from the foil and is discretized using an H-H type grid with a total of 2.1 million grids (191x111x101); 121x81 grid points are used to discretize the foil surface. The first grid above the hydrofoil surface is at \( y^+ = 1 \) in order to properly describe the boundary layer.

We consider an incoming uniform flow at an angle of attack relative to the foil of 8°. Freestream velocities and pressures are specified in the upstream inflow boundary and in the far-field side boundaries, and a first order extrapolation for all variables is used at the outflow boundary. A symmetry boundary condition is applied at the foil root section and no-slip and zero normal pressure gradient conditions are imposed on the foil surface. The flow is directly simulated without the use of a turbulence model in order to capture unsteady flows separation and vortex shedding at a Reynold number \( Re = 1.5 \times 10^6 \) (chord length \( C = 0.15 \) m and liquid velocity \( U_{\infty} = 10 \) m/s).

In the absence of nuclei, unsteady flow separation with vortex shedding is observed in the liquid and the computations are conducted until limit cycle oscillations are reached. Figure 1 shows an example of the non-dimensional pressure contours, \( \bar{p} = (p - p_\infty) / \rho V_{\infty}^2 \), and the iso-pressure surface \( \bar{p} = -0.5 \) at a selected time after reaching limit cycle oscillations.

**Figure 1.** Non-dimensional pressure contours and iso-pressure surface at level -0.5 shown at a selected time after limited cycle oscillations are reached.

**Dispersed Bubbly Flow**

To study the interaction between nuclei and the foil, 90,000 free field bubble nuclei are used with 50,000 nuclei of radius 20 µm, 30,000 of radius 40 µm and 10,000 or radius 60 µm. The nuclei are released from a preset release domain upstream of the foil. The domain’s width (0.32 m) is specified to cover the whole span of the hydrofoil. The height is selected to be 0.04 m in order to cover the “window of opportunity” though which nuclei need to pass in order to encounter the low pressure on the hydrofoil surface and the tip region. The release domain has a length of 1.5m aligned in the streamwise direction such that enough nuclei can be supplied to conduct the unsteady simulations for 0.15 second.. The 90,000 nuclei in this release domain correspond to an initial average void fraction of \( 10^{-6} \). Figure 2 shows the bubble distribution in the foil flow field due to dynamic effects when gas diffusion is not included.

This was for a cavitation number, \( \sigma = 1.0 \), where the cavitation number is defined as:

\[ \sigma = \frac{p_{\infty} - p_c}{0.5 \rho U_{\infty}^2} \] \tag{7} 

We can observe that the nuclei grow to large sizes in the low-pressure regions on the suction side of the foil near the leading edge and in the tip vortex region. However, the bubbles return to their original size once the field pressure returns to the upstream pressure level and no significant effect on the bubble size distribution can be detected.
EFFECTS OF GAS DIFFUSION

Gas Diffusion Model

To study the effect of gas mass transfer in and out of the bubbles on the dispersed bubbly flow, a gas diffusion model is used. This is based on the fact that in the presence of a concentration gradient, dissolved gas will diffuse from high concentration regions to low concentration regions. The transport equation for the gas of concentration, $C(x,t)$, is given by:

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = D_g \nabla^2 C,$$  \hspace{1cm} (8)

where $D_g$ is the gas molar diffusivity.

At a bubble interface, the dissolved gas concentration, $C_s$, is connected to the gas pressure inside the bubble, $p_b$, through Henry’s law:

$$C_s = C_e \text{ at } r = R \quad \text{with} \quad C_e = \frac{p_g}{H},$$  \hspace{1cm} (9)

where $H$ is the Henry constant. This condition at the bubble interface is very important and actually drives the gas diffusion dynamics. The other initial and boundary conditions for gas diffusion are:

$$C = C_e \text{ for } t = t_0,$$
$$C \rightarrow C_a \text{ for } r \rightarrow \infty,$$  \hspace{1cm} (10)

where $C_a$ is the dissolved gas concentration far away from the bubble surface.

The gas transfer rate, $\dot{n}_g$, at the bubble/liquid interface, is directly proportional to the product of the interface area and the normal concentration gradient at the interface,

$$\dot{n}_g = D_g \int_0^1 \frac{\partial C}{\partial n} dS.$$  \hspace{1cm} (11)

To obtain the gas pressure inside the bubble taking into account gas diffusion, we consider the instantaneous energy and mass balance of the bubble content. Both components of the bubble content, vapor and gas, are assumed to be ideal gases and follow the ideal gas law:

$$(p_b + p_v) V_b = (n_v + n_g) R_b T_b,$$  \hspace{1cm} (12)

where $V_b$ is the volume of bubble and $n_i$ is the number of moles of gas within the bubble. $R_u$ is the universal gas constant, and $T_b$ is the absolute temperature of the gas and vapor mixture. One consequence of this assumption is that the ratio $n_g / n_v$ in the bubble is directly proportional to the ratio of the partial pressures, $p_g / p_v$.

Due to the relatively short vaporization time compared to bubble dynamics and gas diffusion characteristic times, the vapor is considered to instantaneously flow into and out of the bubble, and $p_v$ is assumed equal to the equilibrium vapor pressure of the liquid at the bubble wall temperature.

Considering the thermodynamics of the contents of the bubble and applying the first law, the energy balance for the control volume bounded by the bubble surface is

$$dU = -dW + \sum_{i = g, v} \dot{n}_i h_i dt,$$  \hspace{1cm} (13)

where $dU$ is the change in internal energy, $dW$ is the work done on the control volume, and $h_i$ is the specific enthalpy of constituent $i$. The various terms in Eq. (13) can be expanded as:

$$dU = \sum_{i = g, v} c_{V,i} d(n_i T_i),$$ $$dW = (p_g + p_v) dV_b,$$ $$h_i = \sum_{i = g, v} c_{P,i} T_i,$$

where $c_{V,i}$ and $c_{P,i}$ are the specific heat at constant volume and pressure, respectively, and $T_i$ is the liquid temperature. Combining Eq. (12), (13) and (14), and using superscripts $T_b$ and $T_i$ for $c_{V,i}$ and $c_{P,i}$ to indicate whether the specific heats are evaluated at the corresponding bubble or liquid temperature we obtain:

$$c_{V,g} p_g \dot{V}_g + c_{V,v} (p_g \dot{V}_v + p_v \dot{V}_b) + (p_v + p_g) \dot{V}_b$$
$$- c_{P,g} \frac{p_v}{p_g} \dot{n}_g + c_{P,v} T_i \frac{p_v}{p_g} \dot{p}_g - \dot{n}_g c_{P,g} T_i = 0.$$  \hspace{1cm} (15)

Using the fact that $c_P - c_V = R_u$, Eq. (15) becomes:
Integration of Eq. (16) provides the instantaneous gas pressure to be used in Eq. (5) and Eq. (9).

\[ \dot{p}_g = \frac{E / p_g - D p_g - F}{A + B / p_g} , \]

\[ A = \left( c_{p,g}^{T} - R \right) \frac{V_b}{R_u} , \quad B = c_{p,g}^{T} T_i p_i n_g , \]

\[ D = c_{p,g}^{T} \frac{V_b}{R_u} , \quad E = c_{p,g}^{T} T_i p_i \dot{n}_g , \]

\[ F = c_{p,g}^{T} - c_{p,g}^{L} T_i p_i \dot{n}_g . \]  

(16)

Gas Diffusion Effect on Bubbly Flow

Gas diffusion effects on the bubble distribution over the lifting surface are then considered when the water is assumed to be saturated with gas (i.e. with a dissolved gas concentration of 100%, or 0.66 mol/m³). Figure 3 shows the resulting bubble distribution. Here, we see a similar behavior to that in the absence of gas diffusion during the bubble growth phase. However, significant differences are observed in the wake region after the bubbles start collapsing and rebounding. With gas diffusion included, the bubbles fill up with gas and become of much larger sizes than in the absence of gas diffusion. This ends up making the fine vortices in the wake visible due to the collected bubbles.

**Figure 3.** Dispersed bubbly flow over the finite-span hydrofoil when gas diffusion effects are taken into consideration. \( \sigma = 1.0 \). \( \alpha = 8^\circ \).

By observing the behavior of individual bubbles, we are able to demonstrate the effect of gas diffusion on the bubble dynamics better. Figure 4 shows a typical scenario for a small free nucleus of initial radius, \( R_0 = 50 \) µm. As the nucleus encounters a low-pressure region downstream of the leading edge it can either grow modestly and then oscillate or undergo explosive growth and violent collapse depending on the cavitation number. The figure shows that at the higher cavitation number of 1.2 the bubble grows to a maximum bubble radius of 0.65 mm and then reduces size as the pressure along the hydrofoil recovers. In this case, the bubble returns to a size of 70 µm downstream of the foil, which is not much larger than its initial size.

**Figure 4.** Bubble size variations over the finite-span hydrofoil when gas diffusion is taken into account for two cavitation numbers: a) \( \sigma = 1.2 \), and b) \( \sigma = 1.0 \).

At the lower cavitation number of 1.0, the bubble grows explosively to a much larger size and the number of moles of gas, \( n_g \), inside the bubble cumulates significantly over time. As a result, after collapse and several oscillations the bubble ends up with a much larger radius (180 µm) than the original 50 µm radius due to a net increase of the amount of gas in the bubble.
To illustrate quantitatively the effects of the foil presence and of gas diffusion on the bubbly flow as they pass over the finite-span hydrofoil, we sum the volume of all bubbles in volume slices with full height in the \( z \)-direction and with a width, \( \Delta y = 0.32 \text{ m} \) in the span direction, and \( \Delta x = 0.01 \text{ m} \) in the chord direction. Figure 5 shows the total bubble volume in this \((\Delta x, \Delta y)\) volume slice along the streamwise direction, i.e. in the \( x \)-direction.

Figure 5. Effect of gas diffusion on the total bubble volume in volume slices with a width, \( \Delta y = 0.32 \text{ m} \), in the span direction and \( \Delta x = 0.01 \text{ m} \) in the chord direction.

The strong effect of the gas diffusion can be seen downstream of the sheet cavity-like region. Between \( x = -0.02 \text{ m} \) and \( x = 0.08 \text{ m} \) the bubble volume increases dramatically (by almost six orders of magnitude) due to explosive bubble growth on the foil suction side. In this region, the effects of gas diffusion are not visible. This is followed by bubble collapse and successive rebounds and oscillations for \( x > 0.08 \text{ m} \). In this region, the total bubble volume with gas diffusion is seen to become two orders of magnitude larger than that in the absence of gas diffusion. This difference in the total bubble volume is less visible between \( x = 0.14 \text{ m} \) and \( x = 0.25 \text{ m} \) because the volume of bubbles in the tip vortex region is dominant in the volume contribution.

Figure 6 shows a comparison of the bubble number density size distribution at \( x = 0.4 \text{ m} \) with and without gas diffusion. The bubble number density was obtained in a slice volume of \( 0.01 \text{ m} \times 0.32 \text{ m} \times 0.05 \text{ m} \) at \( x = 0.4 \text{ m} \). It is seen that, while without gas diffusion, the bubbles sizes return to the initial radii of 20 \( \mu \text{m} \), 40 \( \mu \text{m} \) and 60 \( \mu \text{m} \), with gas diffusion the foil significantly modifies the bubble size distribution downstream to a wide range between 20 \( \mu \text{m} \) and 300 \( \mu \text{m} \).

**Figure 6.** Effect of gas diffusion on the bubble size distribution at \( x = 0.4 \text{ m} \). Initial bubble distribution was composed of bubble sizes 20 \( \mu \text{m} \), 40 \( \mu \text{m} \) and 60 \( \mu \text{m} \).

**EFFECT OF BOUNDARY NUCLEATION**

A boundary nucleation model based on existing experimental observations and theoretical studies was introduced by Hsiao et al. (2017b) to model the initiation and dynamics of sheet cavitation on foils. In the nucleation model, nuclei are released from the rigid boundaries when the pressure at the center of a discretized cell drops below a threshold pressure, e.g. the vapor pressure. \( N \) nuclei are then released from the cell in the flow field during the time interval \( \Delta t \):

\[
N = N_s f_n \Delta t \Delta A, \quad (17)
\]

where \( N_s \) is the number density of nucleation sites per unit area, \( \Delta A \) is the cell surface area, and \( f_n \) is the nuclei release rate. Hsiao et al. (2017b) applied this boundary nucleation model to simulate sheet to cloud cavitation and found it to recover very well experimentally measured (Berntsen et al. 2001) time-averaged cavity lengths and oscillation frequencies for a range of conditions.

We consider this model here to investigate the effects boundary nucleation have on the bubble distribution downstream of the foil. Figure 7 shows the resulting bubble distribution with only boundary nucleation and both field nuclei and boundary nucleation present. It is seen that when considering only the boundary nucleation, tip vortex cavitation does not develop because pressure at the hydrofoil surface near the tip region does not drop below the threshold to feed nuclei into the tip vortex. The tip vortex cavitation is controlled by the free nuclei coming from a restricted volume upstream of the hydrofoil dubbed the “window of opportunity” (Hsiao & Chahine 2005).
only the free nuclei. This is clear everywhere except between $x = 0.14m$ and $x = 0.25m$ where the total bubble volume is composed of the bubbles captured in the tip vortex. Finally, in the wake region, the number density of larger-sized bubbles obtained from both nuclei sources is much higher than that obtained from the free nuclei only as seen in Figure 9.

Figure 7. Effect of free nuclei and boundary nucleation on the bubble distribution downstream of the foil: a) boundary nucleation only ($f_n = 0.8$ kHz and $N_s = 10$ cm$^{-2}$) and b) both free nuclei and boundary nucleation included. $\sigma = 1.0$, $\alpha = 8^\circ$.

Figure 8. Effect on the bubble volume at different streamwise locations of the two nuclei sources: free field nuclei and boundary nucleation. $\sigma = 1.0$, $\alpha = 8^\circ$.

Figure 9. Comparison of bubble size distribution at $x=0.4m$ between the simulations with free nuclei only, boundary nucleation only and both nuclei sources. $\sigma = 1.0$, $\alpha = 8^\circ$.

EFFECTS OF BUBBLE BREAKUP

In the absence of a precise model for bubble breakup in pressure gradients and near boundaries, Hsiao et al. (2017a) investigated the effects of bubble breakup using a heuristic model with the following characteristic parameters: breakup threshold, breakup moment, number of daughter bubbles, sizes of daughter bubbles, and locations of daughter bubbles. Although inclusion of such bubble breakup model resulted qualitatively in better correspondence with observations and made the fine vortices in the wake more visible due to collected bubbles, the parameters used in the simulations were not based on solid grounds. We attempt here to improve this breakup model.

Model Based on Bubble Stability Analysis

The stability of the bubble shape to non-spherical disturbances has been studied analytically for a long time using perturbation analysis, e.g., (Birkhoff 1954, Plesset & Mitchell 1956). The non-spherical bubble shape can be described as a linear superposition of spherical harmonics:

$$ R(t) = R_0 + \sum a_n Y_n^0, \quad (18) $$

where $Y_n^0$ is a spherical harmonic of degree $n$ and $a_n$ is perturbation amplitude which is a function of time.
The stability of the spherical interface is compromised when perturbation amplitude grows significantly.

For small order perturbations, Plesset & Mitchell (1956) derived the following equations for $a_n(t)$ after substituting Eq. (18) into the Rayleigh-Plesset equation:

$$\ddot{a}_n + \frac{3\dot{R}}{R} \dot{a}_n + Aa_n = 0,$$

$$A = (n-1)(n+1)(n+2)\frac{\gamma}{\rho R^2} - \frac{(n-1)}{R} \dot{R}. \tag{19}$$

Eq. (19) expresses a damped harmonic free oscillation system, which becomes most unstable when $\dot{R}<0$ and $R_0 \geq \dot{R}$ (Brennen 1995, 2002). This leads to a rapid growth of non-spherical distortions and disintegration of the bubble. The growth rate of the distortion is proportional to $A$ and the harmonic mode which leads to the maximum value of $A$ is when $dA/dn=0$, i.e.

$$\frac{dA}{dn} = 3n^2 + 4n - (\Gamma + 1) = 0, \quad \Gamma = \frac{\rho \gamma R^2}{\gamma} \dot{R}. \tag{20}$$

Eq. (20) has a positive real root only when $\Gamma > 0$ and its value is given by:

$$n = \frac{1}{3} \left[ (7 + 3\Gamma)^{1/2} - 2 \right]. \tag{21}$$

Note that $\Gamma$ is time dependent and reaches a maximum at the beginning of each bubble rebound.

Figure 10 shows the time history of a bubble radius and of $\Gamma$ for a typical cavitation bubble traveling along the hydrofoil surface. It can be seen that the value of $\Gamma$ always reaches a local maximum close to the time when the bubble reaches a local minimum size. Therefore, we will impose the bubble breakup moment to be at the time of rebound if it satisfies the breakup criterion.

Brennen (1995) estimated the final number of fragments after many sequences to have a cubic dependence on the mode $n$. Here, the breakup model is applied at each bubble rebound to all bubbles including parent and child. We therefore will use the mode number, i.e. the integer $n$, as the number of daughter bubbles at each breakup.

After breakup, each daughter bubble is also tracked individually and the same model parameters are imposed if the breakup criterion is satisfied again. With this simple assumption, the bubble breakup criterion can be simply determined if $n \geq 2$. We also assume that all daughter bubbles are the same size, and thus have a well-defined radius $R_d$ conserving the bubble volume,

$$R_d = R^n \times 1/3. \tag{22}$$

As a result, we can determine all characteristic parameters based on bubble shape instability analysis and randomly distribute all daughter bubbles within the confines of the parent bubble.

Figure 10. Bubble radius versus time and corresponding $\Gamma$ versus time for a typical cavitation bubble traveling over the hydrofoil surface.

Figure 11. Dispersed bubbly flow over the finite-span hydrofoil with gas diffusion and breakup model based on shape instability considered. Only free nuclei are considered in this simulation. $\sigma=1.0, \alpha=8^\circ$.

Figure 11 shows the resulting bubble distribution when the bubble breakup model is applied and only free nuclei are considered. By comparing to Figure 3, we can see significant differences in the wake region, downstream of the large bubble growth and collapse region, where breakup occurs at rebounds. The visual appearance of the bubbles now is close to that seen experimentally (Russell et al. 2016).
Comparison of the total bubble volumes in Figure 12 does not show a major influence of breakup on the total volumes because of the condition imposed of volume conservation at breakup. However, the bubble size distribution in the wake region is significantly different between the two cases. As can be seen in Figure 13, the breakup model significantly alters the bubble size distribution at x=0.4m. Including bubble breakup results in a significant increase in the number density of small and mid-size bubbles and a reduction in the large size bubbles. This leads to a dominant bubble size of about 100 μm in the wake region.

Model Based on Experimental Observations

Another way to determine the characteristic parameters to use in the numerical model is through experimental observations of cavitation bubble dynamics and breakup. To enable close-up observations of traveling cavitation bubbles and their breakup during collapse, we have setup a closed flow loop with a venturi as the test section. A sketch of the flow loop is shown in Figure 14. Vacuum can be applied to the supply reservoir to control the pressure at the venturi exit. The venturi has a constant diameter throat length of 13.7 mm. The venturi shape was selected to generate a local pressure field similar to that encountered by a bubble near the leading edge of the hydrofoil.

By adjusting either the flow velocity or the reservoir pressure, we are able to control the cavitation number and select it to be slightly beyond cavitation inception such that traveling cavitation bubbles can be observed in the throat area of the venturi. Figure 15 shows a time sequence of images from high speed movies, for two traveling bubbles at a flow velocity of 14.5 m/s at the throat when the reservoir pressure was 1 atm. From this sequence, we can measure the maximum bubble size and count the number of visible daughter bubbles after breakup.

From a set of high-speed videos all taken under the same flow condition we were able to observe a reasonable number of isolated cavitation bubbles growing to a range of different maximum radii and count the number of daughter bubbles after breakup. As shown in Figure 16, with this set of analyzed data, we are able to establish a preliminary correlation between the number of visible bubbles after breakup and the maximum radius the bubble reached in the throat, $R_{max}$:

$$n = 2 + 2.7 \left( R_{max} - R_c \right),$$  \hspace{1cm} (23)

where $R_c$ is the observed critical maximum bubble size below which bubble break does not occur, and where the bubble radii are expressed in millimeters.

Concerning the breakup implementation, we select the bubble breakup moment to be at the time of volume rebound and we randomly distribute the daughter bubbles within the confines of the parent bubble. Using Eq. (23) for the number of daughter bubbles with uniform breakup size and $R_c$ as the
breakup threshold, we apply this breakup model to the foil flow described above and predict the bubble size distribution downstream of the hydrofoil considering again only free nuclei. The resulting dispersed bubbly flow over the finite-span hydrofoil is shown in Figure 17. Although the wake obtained with this empirical breakup model looks similar to that shown in Figure 11, the total number of bubbles and the predominant bubble sizes are quite different as illustrated in Figure 18.

Figure 15. Two time sequences from high speed videos taken at 20,000 fps showing examples of traveling cavitation bubbles at a flow velocity of 14.5 m/s in the throat section and the reservoir pressure equal of 1 atm.

Breakup Model Based on Numerical Simulations

In order to establish a breakup model based on numerical simulations, one should consider 3D simulations of traveling cavitation bubbles. We preliminarily did so using the multiscale two-phase flow model developed by Hsiao et al. (2017b) in order to gain insight into the mechanism leading to bubble breakup.

Figure 16. Number of daughter bubbles following breakup as a function of the maximum bubble radius reached in the venturi throat. Throat flow velocity of 14.5 m/s, reservoir pressure 1 atm.

Figure 17. Dispersed bubbly flow over the finite-span hydrofoil with gas diffusion included and using the breakup model based on the experimental observations conducted on a venturi flow. Only free nuclei are considered in this simulation. $\sigma=1.0$. $\alpha=8^\circ$.

Figure 18. Comparison of the bubble size distribution at $x=0.4m$ obtained using for bubble breakup using the stability analysis model and the laboratory tests empirical model. $\sigma=1.0$. $\alpha=8^\circ$. 
An example result is illustrated in Figure 19. In this simulation, a small nucleus with an initial radius of 20 \( \mu \text{m} \) is released upstream of the hydrofoil. A level-set based cavity model is used to simulate the bubble deformation and motion. As seen in the figure, the bubble grows explosively and strongly deforms while traveling into the low-pressure region on the suction side of hydrofoil. At its maximum size, the height of the bubble reaches 5 mm while the boundary layer thickness is about 0.7 mm. The bubble then breaks up into multiple smaller bubbles as it travels further downstream and encounters higher pressures and shear flow near the foil boundary.

This preliminary simulation implies the important role played by the local vorticity and shows bubble-induced flow disturbance during the bubble dynamics and breakup. In on-going studies, the simulations will be refined with the ultimate goal of uncovering breakup criteria to correlate the breakup parameters with local flow conditions such as vorticity and turbulence, etc.

CONCLUSIONS

An Eulerian-Lagrangian model was used to simulate a dispersed bubble flow over a finite-span rectangular hydrofoil. Gas diffusion during bubble oscillations were included and are seen to have a major effect on the resulting bubble size distribution downstream of the foil, which contains much larger bubbles than upstream. Taking into account boundary nucleation is also seen to affect significantly the distribution with very fine bubbles collecting in the vortices in the foil wake thus visualizing these vortices.

Finally, inclusion of non-spherical bubble collapse resulting in bubble break up into several smaller bubbles is seen to be very important. Bubble breakup results in a significant increase in the number of small and mid-size bubbles and a reduction in the large size bubbles. Two breakup models, one based on spherical shape stability analysis and the other on experimental observations in a venturi flow, were implemented. Both breakup models result in a very significant increase in the number of small and mid-size bubbles and a reduction in the large size bubbles and in bubble distributions which resemble much better available observations.

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