Multiscale Modeling of Cavitation using a Level Set Method with Cavity Detection

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Abstract
An Eulerian-Lagrangian multi-scale two-phase flow model is presented, which includes a micro-scale model for tracking the microbubbles, a macro-scale model for describing the overall flow field and large cavity dynamics, and a transition model to bridge the micro and macro scales. The method also includes a cavity detection scheme which marks each cavity with an ID index so that each can be tracked and treated individually. This approach was implemented in our CFD code 3Dynafs© and used to simulate, for illustration, multiple bubble dynamics simultaneously released at different locations in a Rankine line vortex. The method captures their growth, entrainment into the vortex core, elongation along the vortex centerline, followed by potential splitting and breakup.

Keywords: Multiscale Model; Level Set Method; Cavity Detection

Introduction
Modelling of cavitation is challenging as it involves a wide range of characteristic lengths and phenomena including nucleation, large deforming free surfaces, bubble coalescence, bubble breakup into very fine microbubbles, bubble clouds collective effects, … etc. [1]. Motivated by the need for a physics-based modeling of the dynamics, we are developing a multiscale approach which tracks dispersed microbubbles in a Lagrangian fashion while solving the overall two-phase viscous flow in an Eulerian fashion. The microbubbles are modelled as singularities using a Discrete Singularity Model (DSM) until they grow to no longer be sub-grid or merge into macro cavities or pockets. They are then discretized and fully resolved in an Eulerian grid using a Level Set Method (LSM) [2, 3]. Afterwards, in order to continuously track each individual cavity/pocket, a cavity ID detection scheme [4] is used. This cavity tracking allows to follow each cavity and to assess the properties of its content individually. This capability is particularly highlighted in this presentation, for the simulation of multiple bubble entrainment, as well as bubble breakup in vortex flows.

Multiscale Two-phase Model

Eulerian Continuum Model for Viscous Mixture Flow
The two-phase medium is treated as a continuum, which satisfies the following continuity and momentum equations:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) = 0,$$

$$\rho_m \frac{D \mathbf{u}}{Dt} = -\nabla p + \mu_m \nabla^2 \mathbf{u} + \rho_m \mathbf{g},$$

where \( \mathbf{u} \) is the mixture velocity, \( p \) is the pressure and \( \mathbf{g} \) is the acceleration due to gravity. The mixture density, \( \rho_m \), and the mixture viscosity, \( \mu_m \), are related to the void fraction, \( \alpha \), and to the liquid and gas properties through

$$\rho_m = (1 - \alpha) \rho_l + \alpha \rho_g,$$

$$\mu_m = (1 - \alpha) \mu_l + \alpha \mu_g.$$

\( \alpha \) is deduced from the location and size information of the bubbles using a Gaussian distribution scheme which smoothly “spreads” each bubble’s volume over its surroundings while conserving the total bubble volumes [2].

Lagrangian Discrete Bubble Model for Microbubbles
The bubble model uses a Surface Average Pressure approach [1] to average fluid quantities along the bubble surface and model its dynamics using a Rayleigh-Plesset-Keller-Herring equation:

$$\left[ 1 - \frac{\dot{R}}{c_m} R \dot{R} + \frac{3}{2} \left( 1 - \frac{\dot{R}}{c_m} \right) \dot{R} \right] \frac{\dot{u}_b^2}{4} + \left[ 1 + \frac{\dot{R}}{c_m} R \frac{d}{dt} \dot{R} \right] \left[ \frac{p_l + p_g - p_m - 2 \gamma}{R} - 4 \frac{\dot{R}}{R} \right],$$

where \( R \) is the bubble radius, \( p_l \) is the liquid vapor pressure, \( p_g \) is the bubble gas pressure, \( \gamma \) is the surface tension, \( \dot{u}_b = u_{bc} - u_b \), where \( u_b \) is the bubble travel velocity, while \( u_{bc} \) and \( p_m \) are respectively the liquid velocity and the

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ambient pressure “seen” by the bubble during its travel, and are computed using surface averaged values of the liquid quantities at the bubble interface. $c_m$ is the local sound velocity in the two-phase medium. The bubble trajectory is obtained from the following bubble motion equation [2]:

$$\frac{d\mathbf{u}_b}{dt} = \left(\frac{\rho_b}{\rho_p}\right) \left[\frac{3}{8R} C_D \left| \mathbf{u}_b \right| \mathbf{u}_b + \frac{1}{2} \left( \frac{d\mathbf{u}_{bc}}{dt} - \frac{d\mathbf{u}_b}{dt} \right) \right] - \frac{3R}{2R} \frac{\nabla p}{\rho_l} + \frac{\left( \rho_s - \rho_b \right)}{\rho_l} g + \frac{3C_L \sqrt{R} \mathbf{u} \times \Omega}{4\pi R} \right]$$

(4)

where $\rho_b$ is the bubble contents density, $C_D, C_L$ are the drag and lift coefficients, and $\Omega$ is the deformation tensor.

**Level-Set Method for Macro Cavities**

Once the microbubbles treated as singularities increase size to be able to be resolved in the Eulerian frame, it becomes important to describe their fine details, interface dynamics and deformation. A criterion based on bubble size is set to “activate” the bubbles and define their interface using the level-set approach. Afterwards, based on the initialization distance function field from “activated” bubbles, a Level-Set method is used to simulate large free surface deformations [3]. A smooth function, $\varphi(x, y, z, t)$, whose zero level coincides with all liquid/gas interfaces when the level set is introduced, is defined in the whole physical domain (i.e. in both liquid and gas phases) as the signed distance $d(x, y, z)$ from the interface $\varphi(x, y, z, 0) = d(x, y, z)$. This function is enforced at each time step to be a material surface evolving with the velocity field $\mathbf{u}$:

$$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \mathbf{u} \nabla \varphi = 0.$$  

(5)

In order to continuously track all individual cavities, a cavity ID detection scheme is implemented. This is used to detect, track, and distinguish the properties of the contents of each individual cavity. Each cavity and all of its enclosed fluid cells are assigned a unique ID number, and an ideal gas law is applied independently to each separate cavity.

![Figure 1: Sequences of multiscale method modeling [2] of the cavitation and bubble capture in a line vortex. Rankine vortex of viscous core radius, $a_c=5$mm, and circulation, $\Gamma=1,255m^2/s$. (Top) Bubble deformation; (Bottom) Bubble merging into the developing vapor core. The dark blue spheres are DSM bubbles while the grey iso-surface is the liquid-vapor interface represented by level set method. The grey mesh is the computation grid used by the Eulerian solver and the background color denotes the value of level set at each cell.](image)

**Simulation of Bubble Cavitation in a Line Vortex**

A Rankine vortex is defined by the angular velocity, $u_\theta$, and pressure, $p_\theta$, of the vortical flow given by

$$u_\theta(r) = \begin{cases} \frac{\Gamma}{2\pi a_c^2}, & r \leq a_c; \\ \frac{\Gamma}{2\pi r}, & r > a_c \end{cases}$$

(6)

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where, $r$ is the radial coordinate, $\Gamma$ is the circulation strength and $a_c$ is the viscous core radius. Figure 1 illustrates how the present multiscale method tracks a DSM bubble, which grows large enough to be switched to be represented by the level set free surface. From there on, fine details of the bubbles can be captured. These include its deformation, elongation along the axis of vortex, as well as its merging into the cavity... etc.

**Single Bubble Traveling and Entrainment**

The following simulations show the resolution of bubble shape details obtained by the level set modeling. An initially spherical bubble of a radius of 1mm is released in a Rankine vortex flow with core radius $a_c=5mm$ and a circulation $\Gamma=0.097m^2/s$. The bubble grows and deforms while being drawn into the core center. Figure 2 displays the history of bubble trajectory and shape during this process. It is seen that the present model successfully captures the process of bubble deformation, entrainment and elongation in the vortex, as well as its interaction with the surrounding flow. As seen in the right panel of Figure 2, similar phenomena are also observed in a high speed video taken in the DYNAFLOW lab [5].

![Figure 2](image-url)

*Figure 2: (Left) 3DynaFS\textsuperscript{TM} simulation of the history of a 1mm bubble trajectory and shape while being captured in a Rankine vortex of a core radius $a_c=5mm$ and a circulation $\Gamma=0.097m^2/s$; (Right) Images extracted from high speed video taken in the DYNAFLOW lab [5].*

**Multiple Bubble Tracking**

In this section, the model is used to simulate two bubbles released simultaneously from two field locations in the Rankine line vortex: one on the vortex axis and the other outside of the core. Though the bubbles have the same initial size and gas pressure, due to the different pressures they encounter along their trajectories, they show very different bubble volume oscillations and gas pressure history. The bubble, initially located on the axis, grows to a much larger size immediately after the release. On the contrary, the outside bubble, which experiences much higher pressures, displays strong oscillations of both its volume and gas pressure, even though overall it grows as it moves towards the vortex axis.

It is also interesting to see the dissymmetry in the pressure field in the contour plots in Figure 3, Left due to the interaction between the two bubbles.

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**Bubble Breakup**

In this section we look at bubble breakup modeling. Figure 4 demonstrates bubble elongation along the axis of vortex. As this continues, the bubble breaks up into two separate bubbles with the new daughter bubble given a new ID index. This illustrates the process of bubble tracking and bubble ID assignment.

![Image of bubble breakup](image)

This capability to track generated daughter bubbles is of particular interest to the study of bubble dynamics in generalized and more complex configurations such as on lifting surfaces, where the daughter bubble number and size distributions could play a critical role. For example, in [6] a 3D simulation of a traveling cavitation bubble close to a NACA0015 foil surface, its 3D deformation, and its breakup into multiple smaller bubbles is presented. This is illustrated in Figure 5, which shows a simulation starting with a small bubble released near the leading edge. The small bubble grows large as it travels through the low pressure on the suction side of the lifting surface. It then elongates stream wise while flattening in both the direction perpendicular to the foil surface and that along the span direction. This is particularly highlighted in the lowest image sequence in Figure 5, which displays a zoomed front view of the bubble interacting with the liquid flow around it. This indicates that the bubble elongates in the stream wise direction due to the foil viscous shear flow. At the same time a reverse flow is seen under the bubble, which creates local vorticity and in turn shears off the downstream bubble part, introducing a strong secondary flow.

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underneath. As it travels further downstream and encounters higher pressures, the large elongation and flattening lead the bubble during its collapse to break up into multiple daughter bubbles. Those small bubbles then become new seed nuclei in the viscous wake, which feed the long lived bubbly wake behind the lifting surface [6].

Figure 5: 3D simulation of a non-spherical traveling cavitation bubble on a NACA0015 foil using 3DynaFSS® [6], where the silver iso-surface represents the bubble gas/liquid interface and the background flow is contoured by flow pressure. The bottom row is a front view zoom on the bubble interacting with the liquid flow around it, where the blue iso-surface represents the 3D bubble surface and the purple lines are streamlines in the mid plane.

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In on-going studies, the simulations are being refined to individually identify and quantify the local flow parameters (e.g., shear rate, vorticity etc.) around each daughter bubbles. The ultimate goal is to uncover breakup criteria to correlate the breakup parameters with local flow conditions such as vorticity and turbulence, etc.

Summary

A multi-scale framework is being developed for the simulation of cavitation problem. An example of bubble cavitation in a line vortex is presented to show how the model successfully captures bubble growth, entrainment into the core center to form a gaseous/vaporous core, as well as bubble elongation and breakup in the vortex core. The same model is also applied to simulations of sheet cavity formation and breakup on a foil.

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