Dynamics of a Bubble Cloud near a Rigid Wall

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Abstract. Simulations involving a large number of interacting bubbles in a bubble cloud are conducted to assess the effect of frequency and amplitude of a sinusoidal driving pressure on the bubble cloud behavior. As the pressure amplitude increases, strong nonlinear bubble dynamics become more pronounced and higher pressures are generated at the wall. A resonance frequency, corresponding to the highest collective bubble behavior, results in a pressure peak orders of magnitudes higher than the excitation pressure. This frequency is significantly different from the natural frequency of a spherical cloud executing small amplitude oscillations.

1. Introduction
The collapse of a cloud of bubbles near a rigid boundary is known to be one of the most destructive forms of cavitation. It occurs in a variety of engineering applications including cavitation on propellers, ultrasonic cavitation, cavitating jets, Shock Wave Lithotripsy (SWL), etc. Numerical modeling of the dynamics is very challenging since it involves bubble-bubble, bubble-flow, and bubble-wall interactions [1]. We have examined in [2] several practical simulation models (i.e. not Direct Numerical Simulation) including a continuum homogeneous model, an Eulerian multi-component model, and an Eulerian-Lagrangian model. We found that, although all the models capture the average low-frequency dynamics, the microscale response of the discrete bubbles and the resultant high-frequency local fluctuations are only captured appropriately by the Eulerian-Lagrangian approach. This highlights that discrete bubble modeling is essential to a better understanding of the dynamics since it can resolve the physics down to the single bubble scale, while maintaining computation cost affordable to capture the overall large scale flow behavior. Using a generalized 3D two-way coupled Eulerian-Lagrangian model [3,4], this paper investigates the dynamics of a bubble cloud excited by a surrounding sinusoidal pressure field and computes the resulting pressures on a nearby rigid wall.

2. Numerical Method
The two-phase mixture in the cloud is treated as a continuum with unsteady continuity and momentum conservation equations expressed as follows:

\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) = 0, \quad \rho_m \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu_m \nabla^2 \mathbf{u},
\]

where \(\rho_m\), \(\mu_m\), \(\mathbf{u}\), and \(p\) are respectively the mixture density, dynamic viscosity, velocity, and pressure. \(\rho_m\) and \(\mu_m\) are related to the liquid and gas properties and to the gas volume fraction, \(\alpha\), by:

\[
\rho_m = (1-\alpha)\rho_l + \alpha \rho_g, \quad \mu_m = (1-\alpha)\mu_l + \alpha \mu_g.
\]

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We couple Eq. (1) with those describing the dynamics of the discrete individual bubbles in the cloud. Knowing at each instant all bubble radii and locations provides \( \alpha \) (thus \( \rho_m \) and \( \mu_m \)) as a function of space and substitutes for the need of a mixture equation of state to close the system of equations. Each bubble is treated as a source, which represents volume change, and a dipole to represent translation. The equivalent spherical bubble radius, \( R(t) \), is obtained using a modified Rayleigh-Plesset-Keller-Herring equation [5] which accounts for the mixture compressibility and non-uniform pressure field:

\[
\left( 1 - \frac{\bar{R}}{c_m} \right) \frac{d\bar{R}}{dt} + \frac{3}{2} \left( 1 - \frac{\bar{R}}{3c_m} \right) \bar{R} = \frac{u_e^2}{4} + \frac{1}{\rho_m} \left( \frac{d}{dt} \left( \frac{\bar{R}}{c_m} \right) + \frac{d}{dt} \frac{d}{dt} \right) \left[ p_v + p_R - p_{enc} - \frac{2\gamma}{R} - 4\mu_m \frac{\bar{R}}{R} \right].
\]

\( c_m \) is the local sound speed in the mixture, \( p_v \) is the liquid vapor pressure, \( p_R \) is the bubble gas pressure, and \( \gamma \) is the surface tension. The term \( u_e^2/4 \), accounts for the pressure resulting from the difference between the host medium velocity, \( u_{enc} \), and the bubble velocity, \( u_b \), with \( u_e = u_{enc} - u_b \) [6]. \( p_{enc} \) and \( u_{enc} \) are the encountered pressures and velocities averaged over the bubble surface to account for local non-uniform flow. The bubble trajectory is obtained using the following equation of motion [7]:

\[
\frac{du_b}{dt} = -\frac{3}{\rho_1} \nabla p + \frac{3}{4} C_D \frac{\bar{R}}{R} u_b |u_b| + \frac{3}{2\pi} C_L \left( \frac{\bar{R}}{R} - \mu \right) \frac{u_e \times \omega}{|\omega|} + \frac{3R}{R} u_e.
\]

where \( C_L \) is a lift coefficient, \( \omega \) is the local vorticity, and \( C_D \) is a drag coefficient. The last term is the Bjerknes force due to coupling between bubble volume rate and bubble motion.

3. Problem Setup

As illustrated in Fig. 1, an initially spherical bubble cloud with a radius, \( A_0 \), is driven by a pressure function \( P(t) = P_0 - P_{amp} \sin(2\pi ft) \). The cloud center is initially at a distance \( X_0 \) from a rigid wall and is composed of small bubbles of initial radii \( R_0 \). The bubbles are randomly distributed within \( A_0 \), resulting in a quasi-uniform initial \( \alpha_0 \) within the cloud and all bubbles are initially at equilibrium with the initial pressure \( P_0 \).

4. Driving Pressure Amplitude Effect

The effects of the driving pressure amplitude are illustrated in Fig. 2, which shows snapshots of the bubble cloud dynamics at different instants for \( P_{amp} = 4 \text{ atm} \) and \( 16 \text{ atm} \). The images from top to bottom are for increasing times during the first cycle of cloud oscillation. In both cases, the bubbles in the cloud grow first then collapse in response to the driving pressure. Overall, the cloud collapses in a cascade with the bubbles at the cloud top collapsing first and those on the bottom last. As the driving pressure amplitude increases weak bubble oscillations with small amplitudes transform into strong bubble growths and collapses as evidenced by the changes of the bubble sizes visualized by the colors in Fig. 2. This is made further clear in Fig. 4, which quantitatively compares, the variations of the radius of a bubble close to the wall for increasing amplitudes. For \( P_{amp} = 1 \text{ atm} \), the radius change is negligible. However, for \( P_{amp} = 25 \text{ atm} \), the bubble grows to a maximum radius 2.5 times larger than \( R_0 \) and then collapses to a minimum of about

![Fig. 1: Schematic of the problem of the dynamics of a bubble cloud near a rigid wall.](image)

![Fig. 2: Time sequence of bubble locations and pressures in a bubble cloud driven by pressure excitation with amplitude of 4 atm (top), and 16 atm (bottom). The last frame of each row is a side bottom view of the pressures at the rigid wall.](image)
Fig. 5: Pressure at the wall center created by a cloud excited at different frequencies. \( P_0 = 10 \text{ atm, } A_{\text{amp}} = 9 \text{ atm, } R_0 = 50 \mu \text{m, } A_0 = 1.5 \text{ mm, } \alpha_0 = 5\% \text{ and } X_0 = 1.5 \text{ mm.} \)

5. Driving Frequency Effects

As illustrated in [8], tuning between the cloud characteristic frequency and the pressure field frequency is essential to generating very high collapse impulsive loads. When the driving pressure frequency matches the characteristic frequency of the bubble cloud, strong collective behavior occurs and very high pressures are generated at the wall. We compare in Fig. 5, for the same pressure amplitude of 9 atm, the effect of the driving frequency on the pressure at the wall. As the figure shows, the cloud behavior is very sensitive to the driving frequency. The highest collective effects appear to be at 8 kHz and result in a pressure peak of ~ 300 atm, two orders of magnitudes higher than the excitation pressure. As illustrated in Fig. 6, which shows bubble radii versus time for different bubbles in the cloud and the corresponding pressure variations, the optimum occurs when the driving pressure reaches its highest level while the individual bubbles are collapsing collectively. For lower frequencies, the driving pressure reaches its peak too early and drops before the bubbles start their collapse. Conversely, for higher frequencies, the driving pressure reaches its highest value too late and the bubble volumes change are not in phase. This optimum frequency of 8 kHz, is here much smaller than the natural frequency of a cloud, \( f_{\text{cloud}} \), derived for small amplitude oscillations and linear bubble dynamics theory [9,10],

\[
f_{\text{cloud}} = f_0 \left( 1 + \frac{12}{\pi^2} \frac{A_0^2}{R_0^2} \frac{\alpha_0}{1 - \alpha_0} \right)^{-\frac{1}{2}}, \text{ with } (2\pi f_0)^2 = \frac{P_0}{\rho R_0^2} \left[ 3 - \frac{2\gamma}{P_0 R_0} \right]. \tag{5}
\]

which gives a value of \( f_{\text{cloud}} = 23 \text{ kHz.} \) The discrepancy is due to the large amplitude oscillations and non-linear behavior of interest here.
Conclusions

The dynamics of a bubble cloud subjected to an imposed pressure time variation and the resulting pressure loads on a nearby rigid wall was studied using a 3D two-way coupled Euler-Lagrange model. Strong nonlinear bubble dynamics occur as the amplitude increases and very high pressures are generated on the wall when the cloud collapses. At resonance frequency, a pressure peak orders of magnitudes higher than the excitation pressure occurs and corresponds to a matching between the driving pressure reaching its maximum and the collective individual bubble collapse. Off-resonance, the driving pressure reaches its peak amplitude too early or too late. The non-linear resonance frequency is significantly different from the classical bubble cloud natural frequency.

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