ABSTRACT
This work uses a compressible Eulerian multi-material solver with three modeling approaches to examine shock and pressure wave propagation in a bubbly medium. These approaches represent different levels of complexity from fully resolving the dispersed bubbles to treating the bubbly medium as a homogeneous mixture. An intermediate approach is based on treating bubbles as discrete singularities. Propagation of the pressure wave through the bubbly medium is compared between the simplified approaches and the fully resolved bubble simulation. Different scenarios demonstrating the effect of pressure amplitude, void fraction, and bubble size distribution are presented to further understand wave propagation in bubbly media.

INTRODUCTION
Understanding the propagation of acoustic pressures or shock waves through a bubbly medium is critical in various application areas such as bubble size distribution measurements [1], shock waves and cavitation mitigation [2], cavitation erosion [3], shock wave lithotripsy [4], and initiation of explosives [5].

A considerable effort has been expanded to understand these problems using direct numerical simulations [6]-[11] where the embedded bubbles were fully discretized without any modeling assumptions. Though, these simulations are reliable and accurate, they lack in number of bubbles representing the realistic void fractions due to computational resource limitations.

Here, we present a compressible Eulerian multi-phase multi-material flow solver with three modeling approaches to examine shock wave propagation in a bubbly medium [6].

In a first approach the multi-material solver is used directly and resolves the gas-water interfaces by fully discretizing each bubble. In a second approach an Eulerian Lagrangian approach is considered. A discrete singularity method (DSM) is used to describe the dynamics and motions of the bubbles and their influence on the medium as resolved singularities [12], [15], [16]. Other researchers have used similar approaches for bubbly flow modeling [17] - [19]. In this setting, the bubbles are treated in a Lagrangian manner as spherical entities, which follow a modified Rayleigh-Plesset-Keller-Herring equation and an equation of motion for bubble translation. This model is coupled with the compressible Eulerian solver and can handle a large number of bubbles. The third approach uses a homogeneous equivalent medium approach to describe the two-phase bubbly mixture. This uses directly the compressible Eulerian solver in a medium with space and time evolving void fractions starting with a prescribed initial void fraction distribution.

NUMERICAL METHODOLOGY

Compressible Flow Solver
The present multi-material multi-phase compressible flow solver solves for each material continuity, momentum, and energy equations in a fixed Cartesian grid (Eulerian method):
\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{V}) = 0, \\
\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla (\rho \mathbf{V} \mathbf{V} + p \mathbf{I}) = 0, \\
\frac{\partial E}{\partial t} + \nabla ((\rho E \mathbf{V} + p \mathbf{I}) \mathbf{V}) = 0, 
\]

where \( \mathbf{V} \) is the material velocity vector, \( \rho \) is its density, \( E \) is its specific total energy, and \( p \) is the pressure. The system is closed by using an equation of state which defines the pressure as a function of the specific internal energy and the density. In this work we consider air/water mixtures. An ideal gas equation [13] is used for air

\[
p = (\gamma - 1) \rho e, 
\]

where \( \gamma = 1.4 \) and the specific internal energy, \( e \), is defined as:

\[
e = E - \frac{1}{2} \mathbf{V}^2. 
\]

Tait law [14] is used for water. The expression for Tait law is

\[
p = p_0 + B((\rho / \rho_0)^n - 1), 
\]

where \( n \) and \( B \) are the constants (\( n = 7.15 \) and \( B = 3 \times 10^8 \) Pa for water) and \( p_0 \) and \( \rho_0 \) are a reference pressure and a reference density respectively.

The compressible flow solver uses a high order Godunov scheme employing the Riemann problem to construct a local flow solution between adjacent cells. The numerical method is based on a fully conservative higher order MUSCL scheme [11] to handle material or phase interfaces. This approach uses a Lagrangian treatment to track the interfaces for the cells including more than one materials and an Eulerian treatment for cells away from the interfaces. A re-map procedure is employed to map the Lagrangian solution back to the Eulerian grid. At each computational step, this procedure results in new volume fraction of each material inside the mixed cell. The fluid variables such as density, velocity, pressure, energy, sound speed in the mixed cell are then obtained using average schemes based on the new volume fraction.

This mixed cell approach is here extended to handle a homogenous bubble/liquid mixture in which all the cells containing both air and liquid are treated as mixed cells with the air/liquid interfaces specified according to the initial void fraction distribution [6]. The evolution of the interface in each mixed cell follows the same Lagrangian tracking and remapping procedure as described above.

For the third model considered here, this approach is extended further through coupling with a Discrete Singularities Model (DSM). In this method bubble shapes are not resolved and a bubble is replaced by singularities. In the Eulerian model, a Gaussian scheme is used to define the void fraction due to the presence of the above singularities. Computational cells having a void fraction greater than 0 are treated as mixed cells. However, instead of using the Lagrangian tracking and remapping procedure to update the interfaces and void fraction for each mixed cell, the DSM provides the evolution of the void fraction.

**Discrete Singularities Model (DSM)**

The discrete singularities model is based on a Surface Average Pressure (SAP) approach [12], [15], [16] where the fluid quantities (pressures and velocities) used for the singularity dynamics computations are average quantities over the bubble surface. The source term is obtained from a modified Rayleigh-Plesset-Keller-Herring equation [12], [15], [16] describing the dynamics of the equivalent spherical bubble of radius \( R(t) \),

\[
\left(1 - \frac{R'}{c}\right) \frac{R'}{c} + \frac{3}{2} \left(1 - \frac{R'}{3c}\right) R^2 = \frac{1}{\rho} \left(1 + \frac{R'}{c}\right) \frac{R}{c} \left(p_s - p_{enc} - \frac{2 \sigma}{R} - \frac{4 \mu R'}{R} \right), 
\]

where \( c, \rho, \mu, \sigma \) are the speed of sound, density, viscosity, and surface tension of the mixture, \( p_s \) is the liquid vapor pressure, \( p_s \) is gas pressure inside the bubble, and \( p_{enc} \) is the ambient pressure “seen” by the bubble obtained by averaging the pressure over the bubble surface.

The bubble translation velocity, \( \mathbf{u}_b \), is obtained by integrating the following bubble motion equation:

\[
\frac{d \mathbf{u}_b}{dt} = \left(\frac{\rho}{\rho_b}\right) \left[\frac{C_D}{8R} \mathbf{v_b} \times \mathbf{u}_b \right] + \frac{1}{2} \left(\frac{d \mathbf{V}}{dt} - \frac{d \mathbf{u}_b}{dt} \right) + \frac{3R}{2R} \mathbf{u}_b \\
- \frac{\mathbf{V} + (\rho_b - \rho) \mathbf{g}}{\rho} + 6.44 R^2 \sqrt{\rho_b \mu} \left[\mathbf{u} \times \mathbf{\Omega} \right] 
\]

where \( C_D \) is the drag coefficient and \( \rho_b \) the density of the bubble content. \( \mathbf{\Omega} \) is the local vorticity vector and, \( \mathbf{u}_b \) is the velocity difference between the average liquid velocity, \( \mathbf{V} \), and the bubble translation speed, \( \mathbf{u}_b \),

\[
\mathbf{u}_b = \mathbf{V} - \mathbf{u}_b. 
\]

The first right hand side term in (6) is a drag force. The second and third terms account for the added mass. The fourth term accounts for the presence of a pressure gradient, while the fifth term accounts for gravity and the sixth term is a lift force [21].

The pressure and velocity fields from the compressible solver are used in (5) to find each bubble wall velocity and size and in (6) to obtain each bubble translation velocity.

**Eulerian/Lagrangian Coupling**

The two-way coupling between the Eulerian compressible flow solver and the Lagrangian DSM is realized by the following steps:

1. The bubbles behavior is controlled by the mixture’s local properties, pressure and flow field, and the dynamics and motion of the individual bubbles are based on these quantities.
2. The local properties of the mixture are determined by the bubbles presence, and the spatial void fraction distribution
and local densities are determined by the instantaneous bubble distributions.

3. The flow field adjusts itself to the evolving mixture void fraction distribution while satisfying mass and momentum conservation.

The key of this coupling scheme is the deduction of the void fraction from the instantaneous bubble sizes and locations. A continuous void fraction function is realized by distributing singular bubble volume using a Gaussian function. The Gaussian distribution scheme assigns the “void” contribution, \( a_{i,j} \), of a bubble \( j \) to a nearby cell \( i \) as:

\[
a_{i,j} = \frac{\psi_j^b}{\sum a_{k,j}v_{cell}^i} v_j^b, \tag{8}
\]

where \( \psi_j^b \) is the volume of bubble \( j \), \( x_i \), and \( x_j \) are the coordinate center of cell \( i \) and bubble \( j \) respectively, and \( \lambda \) is a user selected “radius of influence”. Here \( \lambda=1 \) corresponds to radius of influence where 99% of the “void” is distributed within one bubble radius. Since (8) does not guaranty that the total volume of the bubbles is conserved, a cell-volume-weighted normalization scheme is adopted to normalize the “void” contribution as follows:

\[
\bar{a}_{i,j} = \frac{a_{i,j}v_{cell}^i}{\sum_{k} a_{k,j}v_{cell}^i} \psi_j^b, \tag{9}
\]

where \( v_{cell} \) is the cell volume. Since each bubble actually contributes its “void” effect to only a limited number of nearby cells due to the Gaussian function decay, the normalization is computed only for the \( N_{cells} \) cells, which are “influenced” by the bubble \( j \).

To compute the void fraction for the cell \( i \) we then sum up the “void” contribution from all the bubbles \( N_i \) within the “influence range” and divide it by the cell volume, i.e.

\[
\alpha_i = \sum_{j=1}^{N_i} \bar{a}_{i,j} = \sum_{j=1}^{N_i} \frac{a_{i,j}v_{cell}^i}{\sum_{k} v_{k,j}v_{cell}^i}, \tag{10}
\]

RESULTS AND DISCUSSION

Interaction of a Pressure Wave with a Single Bubble

The three approaches described above are first applied to simulate the problem of the interaction of a pressure pulse with an isolated bubble in 3D.

Figure 1 shows the setup for the three approaches. A high pressure region at the left boundary of a rectangular domain is released and moves towards a spherical bubble of radius 300 \( \mu \)m located at \( X=5 \) mm. The length of the domain is 10 mm, with the high pressure region of amplitude 5 MPa in a section 1.5 mm long. The breadth and width of the domain have both a dimension of 4 mm. All side boundaries are prescribed with reflective boundary conditions. The right and the left boundaries are prescribed with non-reflective boundary conditions.

A uniform grid with 0.05 mm sides is used for the DNS approach. However, the DSM and the homogeneous approaches have a uniform grid of 0.05 mm in the X and Z directions and 0.8 mm in the Y direction. This is done to make sure the propagation of wave is captured in a similar fashion for all methods while keeping in mind that the DSM and homogeneous approaches do not resolve the bubble shape. The initial profile of the step function separating the high pressure and low pressure region is shown in Figure 2.

In this study, three values of the Gaussian spreading factor, \( \lambda \) (see (8)) are tested for the DSM approach and one value is tested for the homogeneous model. The variation in void fraction distribution as a function of space for different values of \( \lambda \) is shown in Figure 3. A larger value of \( \lambda \) results in a wider spread of the non-zero void fraction region where the presence of the bubble is taken into account.
Figure 3. Spatial variation of void fraction along the length of domain for different values of Gaussian spread factor, $\lambda$.

Figure 4. Pressure contours showing reflected expansion wave at $t=2\,\mu$s. (a) DNS approach (b) coupled with DSM approach (c) Homogenous model.

The evolution of the solution is shown in Figure 4 through Figure 6. The incoming pressure wave interacts with the bubble resulting in an expansion wave reflecting from the air-water interface. The reflected expansion wave is captured by all three methods and in almost the same way, as can be seen in the representation of the temporal variations of the pressure at $X = 3\,\text{mm}$ (upstream of the bubble) in Figure 5. For $\lambda>1$, where the bubble void fraction is spread over a too large a region, the attenuation of the wave happens much earlier.

On the other side of the bubble, the DSM-coupled model and homogeneous model give very close results. However, DNS appears to show a smaller effect of the bubble on the pressure wave time evolution. Modification of the initial wave, which appears as a pressure time oscillations due to the bubble, is less strong and dies out significantly faster than with the other methods and this requires further analysis and understanding.

Figure 5. Variations of the pressure with time at $X = 3\,\text{mm}$ for an incoming shock wave reflecting on a bubble.

Figure 6. Pressure contours showing attenuation of wave at $t=3\,\mu$s. (a) DNS approach (b) coupled with DSM approach (c) Homogenous model.
Further evolution of the pressure wave showing wave diffraction and pressure attenuation are shown in Figure 6 and Figure 7 for t=3 µs and t=4 µs. The pressure contours show that both DSM approach and homogeneous model give results with similar attenuations while the DNS approach results in less attenuation. This is corroborated by the temporal variation of the pressure at X = 8 mm shown in Figure 8. The figure clearly shows that the time of arrival and the amplitude of the wave after interacting with the bubble is almost the same for both DSM approach and homogeneous model with λ = 1.0 (bubble volume spread over a radial distance provided by this value of λ), while the DNS approach predicts a smaller wave perturbation due to the presence of the bubble. This could be due to the fact that the void fraction is distributed in a region larger than the actual bubble size for both DSM and homogenous approaches. However, both DSM approach and homogenous model are capable of capturing major features of the interaction without requiring very fine grid to resolve the bubble as compared to the DNS approach. This implies that these two approaches are promising for simulating pressure wave interaction with bubbly media where large number of bubbles is involved.

Comparison of DSM-coupled and DNS in a bubbly medium

In this section we compare the DSM-coupled procedure with the DNS calculations. A 3D domain of dimensions 20 mm x 5 mm x 5 mm is used. A mono dispersed bubble distribution with bubble size of 200 µm corresponding to void fraction of 2.23 % is considered. A uniform grid of 40 µm is used for the DNS calculations, while a much coarser grid uniform grid of 0.1 mm is considered for the DSM calculations. A high pressure (0.2 MPa) region from X = 0 mm to X = 1.5 mm region is used as initial pressure distribution condition for the problem. The rest of the domain is at 0.1 MPa with bubbles being in equilibrium with surrounding fluid. Here, just for testing, the side boundaries and the left boundary are prescribed with reflective boundary conditions. The right boundary is prescribed with non-reflective boundary conditions.

The temporal variations of the pressure at three locations (X = 1.5 mm, 5 mm, and 10 mm) are compared between the two models and are shown in Figure 10. The pressures at X = 1.5 mm are seen to combine initial propagation of the wave, followed by a reflected strong expansion wave from the bubbly medium. This curve clearly indicates that the DSM model can capture the non-linear expansion wave very well. The
amplitudes of the pressure at $X = 5 \text{ mm}$ and $X = 10 \text{ mm}$ clearly indicate attenuation of the wave. The presence of the bubbles results in transformation of the planar wave into a less coherent wave. This preliminary comparison indicates that the DSM model behaves for $\lambda = 1$ very similarly to the DNS model.

Figure 10. Variation of the pressure at three different locations ($X = 1.5 \text{ mm}$, $5 \text{ mm}$ and $10 \text{ mm}$) obtained with the DSM-coupled model and the DNS model for a shock wave of amplitude 0.2 MPa propagating through a bubbly medium of initial void fraction 2.23%.

Comparison of DSM-coupled and homogeneous model in a Bubbly Medium

In this section we use the compressible solver coupled with either the DSM model or the homogenous approach to study wave propagation in a bubbly medium.

The setup for the problem is shown in Figure 11 with a 3D domain of dimensions $20 \text{ mm} \times 5 \text{ mm} \times 5 \text{ mm}$. All the boundaries except the right boundary are prescribed with reflective boundary condition. The right boundary is prescribed with non-reflective boundary conditions. A base case is created where a pressure wave of strength 0.2 MPa propagates through a bubbly medium composed of bubble distributed between $X = 3 \text{ mm}$ and $X = 18 \text{ mm}$ representing a void fraction of 0.07%.

For the homogenous model, the initial void fraction distribution is specified to be 0.07% uniformly inside the whole bubbly region.

![Figure 11. A representative configuration showing randomly distributed bubbles. A shock wave of a given strength propagates from left to right.](image)

For the DSM approach, the initial void fraction distribution is provided by the number of bubbles and by the bubble size distribution. Here, the medium is defined using mono-dispersed bubbles of size 50 µm, however the approach applies as well with no modifications to poly-dispersed media. For a mono-dispersed condition, the total bubble number, $N$, required for the case can be determined by

$$N = \frac{3\alpha \varphi}{4\pi R^3}, \quad (11)$$

where $\varphi$ is the total volume of the bubbly region considered. These bubbles are then randomly distributed inside the two-phase region. The initial void fraction distribution is then computed using (11) with $\lambda = 3$. This value of $\lambda$ is selected for the comparisons as a higher value of $\lambda$ approaches the homogeneous model. A systematic study for value of $\lambda$ will be done in future work.

Below the pressure amplitude, the void fraction, and the bubbles initial radii are then varied systematically to study their effects on wave propagation and attenuation.

Effects of the Pressure Amplitude

This section illustrates the effects of changing the amplitude, $P_A$, of the pressure when the bubble size and the void fraction are held the same. As the amplitude increases the bubbles are further compressed from the original equilibrium size. This is shown on the bubble radius (Figure 12) for a bubble located at $X = 3 \text{ mm}$. Similarly, the period of the oscillations of the bubbles is reduced as expected from the Rayleigh time, which depends on the square root of the inverse of $P_A$.

The temporal variation of the pressure in front of the bubbly medium (i.e. in the liquid at left at $X = 2 \text{ mm}$) is shown in Figure 13(a). The initial interaction of the prescribed pressure with the reflection from the rigid boundary on the left results in a quasi-rectangular peak. The interaction of the rectangular wave with the bubbly medium generates an expansion wave moving to the left in the liquid. The strength of the expansion wave increases with of the amplitude of the incoming wave, $P_A$, as seen for both approaches in Figure 13(a). Note that the strength of the expansion wave is higher for the homogeneous model for all the cases as it is prescribed with a fixed void fraction.
Figure 12. Variation of the equivalent bubble radius with time for a bubble located at $X = 3$ mm for different pressure amplitudes and for the same initial bubble radius and the same void fraction.

The temporal variations of the pressure at $X = 18$ mm (Figure 13 (b)) shows the effects of the bubbles as the wave traverses the bubbly medium. The arrival time of the waves as computed by the homogeneous model and DSM does not change much with the change in the pressure amplitude. However the wave arrives much later (at 36 µs) for the homogeneous model then in the DSM approach (at 19 µs). Obviously, assuming the medium to be homogenous and with the same initial void fraction has resulted in the known slow phase speed in a mixture, while treating the bubbles and the liquid independently allowed parts of the wave to propagate practically at the liquid speed and arrive early at the observation location.

The variation of the amplitude of the first peak with the pressure amplitude is shown in Figure 14. The figure clearly shows a linear variation of the amplitude of first peak with the pressure amplitude.

Effects of the Initial Void fraction

This section shows the effects of changing the void fraction when the initial bubble sizes and the pressure amplitude are held the same. The variations of the bubble radius as a function of time for a bubble located at $X=5$ mm are shown in Figure 15. The figure shows that the bubble size is compressed less as the void fraction of the bubbly medium is increased for a pressure wave of same amplitude. Clearly, the pressure wave is attenuated more as the void fraction increases. In addition the oscillations appear to contain additional frequencies than the natural frequency of the isolated bubbles indicating strengthening of the interactions between bubbles.

Figure 13. Effect of the amplitude of the imposed pressure wave on the pressures in a bubbly medium at: (a) $X=2$ mm (b) $X=18$ mm. Initial void fraction of 0.07% and initial mono-dispersed bubble sizes of 50 µm.

With the increase in void fraction, the reflected expansion wave from the bubbly medium becomes stronger as shown in the pressures in Figure 16(a) measured in the liquid at $X=2$ mm upstream of the bubbly medium. This is logical as in the extreme limit where $\alpha=1$ (free surface case) full reflection occurs and the expansion wave would have the same but negative amplitude as the incoming wave.

The results also indicate the expected reduction in the wave speed through the bubbly medium [22] due to the increase in the void fraction for both the models. This is expressed by the later arrival of the pressure signal as shown in Figure 16(b) for the location $X = 18$ mm. The figure also indicates increase in attenuation with increase in void fraction for both the models.
Figure 14. Influence of the prescribed shock strength on the peak pressure amplitude measured after propagation through the bubbly medium.

Figure 15. Time variations of the equivalent bubble radius for a bubble located at X = 5 mm for different void fractions and the same shock strength and bubble size.

To quantify better how the void fraction affects the amplitude of the wave at X = 18 mm, the first pressure peak shown in Figure 16(b) is deduced and plotted versus void fraction in Figure 17. As in the previous case, the homogeneous model shows relatively lower values of the peak amplitudes than the DSM model, indicating a higher attenuation.

Effect of bubble size
This section shows the effect of changing bubble size when pressure amplitude and void fraction are held the same.

Figure 16. Effect of the void fraction on the pressures in a bubbly medium with an incoming shock strength of 0.2 MPa and initial mono-dispersed bubble sizes of 50 µm) at: (a) X=2 mm (b) X=18 mm.

Figure 17. Variation of the pressure amplitude peak after propagating through the bubbly medium as a function of the prescribed void fraction.
As indicated in Figure 18 (a) an increase in bubble size does not show a trend in the reflected expansion wave as fewer and fewer bubbles are required with an increase in bubble size. This results in a diminution of these cumulative effects at a location in front of the bubbly medium. The change in bubble size while the void fraction is the same has no effect on the sound speed as in all the cases the pressure wave arrives at the same time at $X = 18$ mm i.e. at $19 \mu$s as seen in Figure 18 (b). However, the arrival time for the homogeneous medium case is longer ($36 \mu$s) as observed in all the scenarios shown above.

The variation of the first pressure peak at $X = 18$ mm is plotted versus bubble size for the DSM approach and is shown in Figure 19. The figure indicates a decrease in the peak amplitude as the bubble size is increased for the same void fraction. It is important to note that one cannot use the homogenous model to conduct such a bubble size dependent study for pressure wave propagation through a bubbly medium. The information about the bubble size effect on the pressure wave attention usually is very important for study the shock wave mitigation and acoustic wave insulation problems.

**CONCLUSIONS**

Three approaches to study pressure wave propagation through a bubbly medium were presented. The DNS compressible code completely captures the bubble dynamics and wave structure. However, it is limited by the numbers of bubbles that can reasonably be discretized. Moreover, this method requires a good resolution for embedded bubbles leading to large computation times making it challenging for real applications with large number of bubbles. Another extreme case approach is to use a homogeneous model to solve the same problem. This method prescribes an initial void fraction (ratio of bubble volumes to the total volume of the layer) corresponding to the bubble region. As an intermediate approach, the compressible flow solver was coupled with a Discrete Singularities Method (DSM) to simulate the same problem. The DSM approach defines bubbles as Lagrangian particles and can capture features like wave scattering and attenuation important for problems of interest to this work. Both DSM approach and homogenous model were shown to reasonably capture pressure wave interaction with a single bubble even without very fine grid to resolve the individual bubble as compared to DNS approach. Both were also used to study propagation of shock wave in a bubbly media and were shown to capture the essential physical behavior. Increase in attenuation and in wave arrival times were observed with an increase in the void fraction. The effect of changing incoming wave pressure indicated minor changes in wave arrival times. However an increase in the amplitude of the incoming pressure wave results in an enhancement in bubble oscillations amplitude. The bubble size has an effect on the propagating wave pressure amplitude, which decreases with an increase in bubble size. However the wave arrival times are not influenced by an increase in bubble size. The homogeneous model was
shown to behave in similar fashion except the wave arrival
times were higher and pressure amplitudes were lower for all
the cases.

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