Experimental and Numerical Investigation of Single Bubble Dynamics in a Two-Phase Bubbly Medium*

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ABSTRACT

The dynamics of a bubble in a dilute bubbly water-air mixture is investigated experimentally and the results compared with a simple homogeneous compressible fluid model in order to elucidate the requirements from a better advanced numerical solution. The experiments are conducted in view of providing input and validation for an advanced bubbly flow numerical model we are developing. Corrections for classical approaches where in the two-phase flow modeling the dynamics of individual bubble is based on spherical isolated bubble dynamics in the liquid or an equivalent homogeneous medium are sought. The main/primary bubble is produced by an underwater spark discharge from charged capacitors, while the bubbly medium is generated using electrolysis. The size of the main bubble is controlled by the discharge voltage, the capacitors size, and the ambient pressure in the container. The size and concentration of the fine bubbles is controlled by the electrolysis voltage, the length, diameter, arrangement, and type of the wires, and also by the pressure imposed in the container. This enables parametric study of the factors controlling the dynamics of the primary bubble and development of relationships between the primary bubble characteristic quantities such as achieved maximum bubble radius and bubble period and the characteristics of the surrounding two-phase medium: micro bubble sizes and void fraction. The dynamics of the main bubble and of the mixture is observed using high speed video photography. The void fraction of the bubbly mixture in the fluid domain is deduced from image analysis of the high speed movies and obtained as a function of time and space. The interaction between the primary bubble and the bubbly medium is analyzed using both field pressure measurements.

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and high-speed videography. Parameters such as the primary bubble energy and the bubble mixture density (void fraction) are varied, and their effects studied. The experimental data is then compared to a simple compressible fluid medium model which accounts for the change in the medium properties in space and time. This helps illustrate where such simple models are valid and where they need improvements. This information is valuable for the parallel development of an Eulerian-Lagrangian code, which accounts for the dynamics of bubbles in the field and their interaction.

**NOMENCLATURE**

- $\alpha$: Void fraction
- $c_b$: Sound speed on bubble surface
- $c_l$: Sound speed of liquid
- $\gamma$: Surface tension at bubble-liquid interface
- $h$: Enthalpy difference between the liquid at pressure $p$ and at pressure $P_\infty$
- $k$: Polytropic gas constant
- $\mu$: Viscosity of the medium
- $p$: Pressure in fluid domain
- $P_b$: Pressure at bubble wall
- $P_{amb}$: Ambient pressure
- $p_g$: Bubble gas pressure
- $p_v$: Vapor pressure
- $R$: Bubble radius
- $R_{max}$: Maximum bubble radius
- $\rho$: Density of the two-phase medium
- $\rho_l$: Liquid density
- $t$: Time
\( \ddot{u} \) Fluid velocity

\( u \) Radial component of fluid velocity

\( \phi \) Velocity Potential

I. INTRODUCTION

The dynamics of a spherical bubble in a liquid has been studied extensively since the early work of Rayleigh [1]. When bubble nuclei in a liquid or gas bubbles encounter a large pressure drop, they can grow explosively and then on encountering a high pressure, they collapse violently generating strong usually negative effects such as damage to nearby boundary, noise, and decay of performance of an operating machinery. For an ideal spherical isolated bubble and in the absence of gravity effects or nearby objects, the dynamics remains spherical, and the bubble volume oscillations can be predicted analytically. Rayleigh [1] analyzed the collapse of an empty spherical cavity and found the time the cavity requires to collapse was:

\[
\tau_{\text{collapse}} = 0.915 R_{\text{max}} \sqrt{\frac{\rho}{P_{\text{amb}}}},
\]

where \( R_{\text{max}} \) is the initial maximum radius of the cavity, and \( \rho \) and \( P_{\text{amb}} \) are the density and pressure of the ambient fluid. The effect of vapor pressure, non-condensable gas, surface tension, and a time-varying ambient pressure were later included by Plesset to produce the Rayleigh-Plesset equation [1-2], which provides a relationship between the oscillating bubble radius, its derivatives, and the imposed pressure function. For given conditions, the full dynamics of the bubble oscillations can be then predicted from integration of this equation. Using the unsteady Bernoulli equation, one can then determine the pressure in the liquid around the bubble.

Rayleigh-Plesset equation, however, ignore the compressibility of the liquid or the medium surrounding the bubble. Compressibility effects are however very important for very strong bubble dynamics as in underwater explosions [2-4] or when the considered bubble is itself surrounded by many other bubbles making the effective medium very compressible. Herring [5], Kirkwood and Bethe [6], and Gilmore [7] and many others since then [8,10] have extended the spherical bubble dynamics equations to include the compressibility of the liquid. More recently, numerical codes to study the behavior of bubble in compressible liquids, including when large non-spherical deformations are involved, have been developed and have been shown to be accurate.
When the compressibility of the medium surrounding the bubble is provided mainly by the presence of a bubbly mixture, much less fundamental work has been published besides studies using homogenous mixture approaches as in [7,11,12]. This applies to both when the studied bubble is just one of the bubbles composing the two-phase medium, or in the simpler case where the concerned bubble is much larger than the finer bubbles forming the surrounding medium and is the driving force to the dynamics of this medium. In both cases, modeler resort to simplifying assumptions, with need for validation, and would benefit from a basic fundamental experimental and numerical study combining experimental observations and numerical / analytical simulations. This is more so since almost all simulation models use some form of single bubble dynamics as the building block to the two-phase models [8,12].

To better understand the dynamics of bubbles emitting strong pressure waves in a two-phase bubbly medium, we set-up in this study a canonical experiment. The experiment considers the dynamics of a primary relatively large bubble (compared to the surrounding bubbles) in a mixture of water and very fine bubbles. For ease of visualization, a hemispherical bubble is spark-generated at a rigid transparent wall. Using the principle of images and ignoring viscous effects in a very thin layer near the wall, this set-up models a spherical bubble in a bubbly medium of infinite extent. More details of the set-up are discussed in the following section. The schematic of the geometry along with the parameters involved is shown in Figure 1.

Figure 1. Definition sketch of the parameters of the problem of a primary bubble in bubbly medium.

This study of the dynamics of a primary relatively large bubble (compared to the surrounding bubbles) in a two-phase mixture is conducted as a combined experimental and numerical task. This paper addresses the experimental part of the study and compares the results with a model based on Gilmore’s model [7] and Brennen’s
bubbly medium equation of state [8] in order to establish the limits of validity of simple homogeneous models and establish requirements for a two-phase Eulerian-Lagrangian code we are developing in parallel and which is the subject a sister paper [13].

The objectives of the present studies are as follows:

a. Experimentally investigate the dynamics of a primary bubble in two-phase mixture by performing experiments encompassing a wide range of cases.

b. Characterize the bubble evolution for different void fractions and bubble energy.

c. Measure bubbly media effect on pressure propagation.

d. Develop and valuate the limits of validity a simple spherical bubble model in a compressible medium composed of a homogeneous bubbly medium.

II. EXPERIMENTAL SET-UP

An effective method to study the dynamics of cavitation or underwater explosion bubbles is the use of electric spark generated bubbles in a container where the ambient pressure can be controlled. Spark generators have been used for studying bubbles in a liquid for quite a long time since the early work of Ellis [15-20]. DYNAFLOW, INC. has several test chambers that can be used to simulate scaled bubble dynamics, and is equipped with flow diagnostics tools including high speed photography and pressure measurement capabilities.

The use of high speed cameras to photograph spark-generated bubbles produces high quality observations of bubble dynamics including clear visualizations of reentrant jet formation inside the bubble. This combined with high frequency pressure measurements can be extremely useful in developing and validating numerical models.

a. Spark Bubbles Generation

Spark generated bubbles at DYNAFLOW are generated by the underwater discharge of a high-voltage charge between two coaxial electrodes contained within a transparent cell. The underwater spark discharge is a process by which electrical energy is converted into small volume plasma which has temperatures as high as 20,000 K, and
pressures as high as $10^9$ Pascals [16]. As the trigger pulse breaks down the hold-off gap between electrodes, current flows between the electrodes with energy being absorbed in the plasma at a rate depending upon the power input and the inertia of the water. Eventually, due to a decrease in the voltage and an increase in the ionization, temperature, and pressure of the plasma, the spark is extinguished. The energy is initially stored in the plasma in the form of dissociation, excitation, ionization, and kinetic energy of the constituent particles. While the heated and pressurized plasma will tend to expand, the inertia of the outside water will tend to confine it. Mechanical work, light radiation, thermal radiation, and thermal conduction dissipate the energy from the plasma at a rate slower than energy input from the spark [16].

Because of the high pressure, the liquid near the plasma interface is initially compressed. This high pressure leads to the formation of a shock wave that radiates outward. The energy in the shock wave comprises 20 to 50 percent of the energy imparted by the spark into the water [16]. After emission of the shock wave, the pressure in the gas sphere quickly falls, but remains well above that of the surrounding liquid. The pressure difference, however, is small enough that compressible effects can be neglected. From this point on, incompressible hydrodynamic effects dominate. The pressurized gas expands into a large bubble which subsequently collapses and re-expands. The phenomena preceding the incompressible hydrodynamic phase occur quickly and are accompanied by light generation that prevents accurate determination of the initial bubble radius using conventional high-speed photography.

The bubble period is modified by controlling the pressure above the free surface in the test cell. Decreasing the value of $P_{cell}$ increases $R_{max}$ and thus, based on (1), increases the bubble period because of both a larger $R_{max}$ and a smaller $P_{cell}$. Increasing the bubble radii and slowing down the bubble collapse helps in obtaining better visualization, which in turn help producing high fidelity data. Larger bubble period enables better data sampling rate in data acquisition, i.e., more data points can be used to represent the quantity recorded.

Another way to increase the bubble size, $R_{max}$ is to increase the energy at spark discharge. The energy of the bubble is controlled by adjusting the voltage supplied to the capacitor of the spark generator and/or by modifying the capacitor.
When the bubble is significantly distant from boundaries, it retains an almost spherical shape through the time of its maximum volume and a radius could be easily measured. In cases where the bubble is distorted by the presence of boundaries, an equivalent radius based on the measured projected area can be used. Here, it was obtained for each case by assuming the bubble to be axisymmetric, and calculating its volume as the volume of a solid of revolution of the observed outline around its vertical axis.

The pressure at the spark generation (bubble center) location was measured using the absolute pressure in the vacuum cell and the hydrostatic head computed using the depth of the electrodes. The vapor pressure was deduced from the measured ambient water temperature.

**b. Bubbly Media generation**

To generate the bubbly medium a wire mesh electrolysis set-up was used. Electric current was applied across the wire mesh to dissociate the water into oxygen and hydrogen. Increasing the current increases the number of microbubbles produced. The microbubble size distribution depended upon the wires diameter, the density distribution of the wire mesh and the ambient pressure inside the spark tank.

*Figure 2. Schematic of a DYNAFLOW's spark cell set-up.*
c. Void Fraction Measurement

The transparent walls allowed recording of the bubble dynamics by a high-speed digital camera (Redlake Imaging model PCI8000S) capable of framing rates up to 62,000 frames/second. From the high-speed movies, the initial and time-varying bubble size distribution and their corresponding void fraction were computed. The images from the high-speed movies were processed using the image processing software - ImageJ [14]. Shown in Figure 4 is the 0.5cm x 0.5cm imaging window selected to compute the void fraction.

Figure 4. Top: An instantaneous snap-shot from the high speed movie. The image window is 0.5cm x 0.5cm (100 x 100 pixels).

Right: The binary image showing the bubbles in the focal plane.
The acoustic signals of the bubbles were measured with a quartz transducer having a 1µs rise time and resonance frequency ≃ 430 kHz (PCB Piezotronics model 102A03).

III. NUMERICAL STUDY

In order to interpret the experimental results within the frame of a classical two-phase continuum model and to evaluate this model, we have considered the following approach. The present numerical study applied to the dynamics of the primary bubble in consideration is based on the analytical second order ordinary differential equation for the bubble wall-motion derived by Gilmore [7]. It applies to a spherical bubble in a compressible medium. Here, we combine this equation with an equation of state which describes the compressibility effects of the surrounding bubbly medium. This medium is assumed to be a continuum with properties uniquely determined by the knowledge of the gas void fraction at each location and time. These properties are taken into account through an equation of state (EOS) describing the pressure-density relation that exists in a bubbly medium [8]. This EOS implicitly accounts for the field microbubble response to the local pressures. This paper aims at examining the validity of the following statement: “Integration in time of the Gilmore differential equation while accounting for the EOS of the surrounding liquid should provide a first order approach to the problem solution."

a. Gilmore Bubble Dynamics Equation

With the assumption of spherical symmetry the flow around a growing/collapsing spherical bubble is irrotational and the continuity and momentum equations can be expressed as follows:

\[ \nabla \ddot{u} = -\frac{1}{\rho} \frac{D\rho}{Dt}, \]

\[ -\frac{\partial \phi}{\partial t} + \frac{u^2}{2} = -\int_{\rho_{\infty}}^{\rho} dp. \]

In the above equations, \( u, \rho, \phi, p \) are functions of the spatial location, \( r \), and time, \( t \). \( \ddot{u} \) is the fluid velocity vector in the radial direction, \( u \) is its amplitude, \( \phi \) is the corresponding velocity potential, \( \rho \) is the local density of the compressible medium, \( p \) is the local pressure and \( p_{\infty} \) is the pressure at infinity. Equation (3) is derived for a
barotropic fluid, i.e. with the assumption that the density of the medium can be expressed as a function of the pressure only. It is convenient to denote the integral appearing on the right hand side of (3) by the symbol $h$:

$$h(p) = \int_{p_\infty}^{p} \frac{dp}{\rho}$$  \hspace{1cm} (4)

Thermodynamically, $h(p)$ is the enthalpy difference between the liquid at pressure $p$ and at pressure $P_\infty$.

Furthermore, if the flow field in the liquid consists entirely of “outgoing” pressure and velocity waves then the well-known expression for diverging spherical sound waves can be applied to the velocity potential [6]:

$$\phi = \frac{1}{r} f \left( t - \frac{r}{c + u} \right)$$  \hspace{1cm} (5)

c in the above equation is the local sonic speed. By plugging the above expression in (3), it can be shown that the quantity $r \left( h + \frac{u^2}{2} \right)$ is also propagated outward with a velocity $c + u$. In other words, the quantity $r \left( h + \frac{u^2}{2} \right)$ follows the wave equation:

$$\frac{\partial}{\partial t} \left[ r \left( h + \frac{u^2}{2} \right) \right] = -(c + u) \frac{\partial}{\partial r} \left[ r \left( h + \frac{u^2}{2} \right) \right].$$  \hspace{1cm} (6)

Manipulating the above equation along with the continuity and momentum equations to eliminate the derivatives with respect to $r$, leads to a particle-derivative relation which when applied at the surface of the bubble ($r = R$), results in the equation governing the interface dynamics of a spherical bubble in a compressible liquid [7]:

$$\left( 1 - \frac{\dot{R}}{c_b} \right) \ddot{R} R + \frac{3}{2} \left( 1 - \frac{\dot{R}}{3c_b} \right) \dot{R}^2 = H \left( 1 + \frac{\dot{\hat{R}}}{c_b} \right) + \frac{R \dot{H}}{c_b} \left( 1 - \frac{\dot{\hat{R}}}{c_b} \right).$$  \hspace{1cm} (7)

In the above equation derived by Gilmore, $R$ is the radius of the spherical bubble, $c_b$ is the sound speed at the bubble surface and $H$ is the enthalpy difference in the compressible medium between the bubble surface and the medium at infinity:

$$H = \int_{P_b}^{p_\infty} \frac{dp}{\rho}$$  \hspace{1cm} (8)

where $P_b$ is the pressure in the liquid at the bubble wall, which can be related to the pressure inside the bubble and the surface tension $\gamma$ by:
\[ P_b = p_v + p_g - \frac{2\gamma}{R}. \]  \hspace{1cm} (9)

\( p_v \) is the vapor pressure of the medium/liquid and \( p_g \) is the pressure of the gas in the bubble. For water at ambient temperature and for cavitation or underwater explosion bubbles vaporization occurs on a very short time scale so that \( p_v \) can be assumed to remain constant during the bubble dynamics.

On the other hand gas diffusion is relatively slow and the amount of gas inside the bubble can be assumed to remain constant so that the gas obeys a polytropic compression law [8]:

\[ p_g = p_{g0} \left( \frac{R_0}{R} \right)^{\frac{3k}{n}}, \]  \hspace{1cm} (10)

where \( k \) in the above equation is the polytropic gas constant and \( p_{g0} \) is the initial gas pressure inside the bubble – corresponding to a bubble radius of \( R_0 \).

If an equation describing the pressure-density relationship for a medium is known, the above equations can be solved to obtain the variation of bubble radius with time.

**b. Expression of Enthalpy in a Liquid**

For pure water, the pressure-density relation for isentropic compression can be given by the following Tait expression [9]:

\[ \frac{p + B}{P_\infty + B} = \left( \frac{\rho}{\rho_\infty} \right)^n, \]  \hspace{1cm} (11)

where \( \rho_\infty \) in the above expression is a reference density corresponding to a pressure \( P_\infty \). \( B \) and \( n \) are known constants for water (\( B \approx 3000 \text{ atm} \), \( n \approx 7 \)). Using (11), the sound speed at a point in the fluid domain can be expressed as follows:

\[ c^2 = \frac{dp}{d\rho} = \frac{n(p + B)}{\rho}, \]  \hspace{1cm} (12)

and the sound speed at the bubble surface, \( c_b \) is given by:
\[ c_b = c_\infty \left( \frac{P_b + B}{P_\infty + B} \right)^{\frac{n-1}{2n}}. \]  

(13)

In the above equation, \( c_\infty \) is the reference sound speed corresponding to the reference pressure \( P_\infty \).

Similarly, using (8) and (11), one obtains the following expression for \( H \):

\[ H = \frac{n(P_\infty + B)}{(n - 1)P_\infty} \left( \frac{P_b + B}{P_\infty + B} \right)^{\frac{n-1}{n}} - 1 \]

(14)

As mentioned earlier, once \( H \) and \( c_b \) on the bubble surface are known, the system of equations describing the wall-motion dynamics of a spherical bubble can be solved.

c. Expression of Enthalpy in a Bubbly Medium

The two-phase medium, composed of water of density \( \rho_l \) and gas of density \( \rho_g \), is characterized by the distribution of void fraction, \( \alpha \), in the domain. The equivalent continuum medium has a density defined as:

\[ \rho_m = (1 - \alpha)\rho_l + \alpha \rho_g \]

(15)

The sound speed in a compressible bubbly medium can be expressed by the following expression derived in [8]:

\[ c^2 = \frac{dp}{d\rho_m} = \frac{p}{\rho_l} \left[ 1 + \frac{\alpha}{1 - \alpha} \right]^2 \left[ \frac{1 + \alpha}{k(1 - \alpha)} + \frac{p}{\rho_l c_l^2} \right]^{-1}, \]

(16)

where \( c_l \) in relation (16) is the sound speed in the pure liquid.

In deriving the above relation, it was assumed that:

- the density of the gas, \( \rho_g \), is negligible relative to the density of the liquid, \( \rho_l \),

- the disperse gas phase obeys the polytropic compression law (\( k \) being the polytropic gas constant),

- surface tension is negligible at the interface of the two components (\( p \) varies smoothly in the medium) and

- the void fraction, \( \alpha \), is small.
Relation (16) can be integrated to obtain an equation of state (relating pressure and density/void-fraction) for the bubbly medium [8]:

\[
\frac{\alpha}{1 - \alpha} = \left[ \frac{\alpha_0}{1 - \alpha_0} + \frac{k}{k + 1} \frac{p_0}{\rho_l C_l^2} \right]^{1/k} - \frac{k}{k + 1} \frac{p}{\rho_l C_l^2}.
\] (17)

In the above equation, \(\alpha_0\) is a reference void-fraction corresponding to a reference pressure \(p_0\). In our computations, we have assumed \(p_0 = P_{\infty}\). Equation (17) can be re-written as follows:

\[
\frac{\alpha}{1 - \alpha} = f(p) = \left( A + D \right) \left( \frac{p_0}{p} \right)^{1/k} - D \frac{p}{p_0},
\]

\[
A = \frac{\alpha_0}{1 - \alpha_0}, \quad D = \frac{k}{k + 1} \frac{p_0}{\rho_l C_l^2}.
\] (18)

Then \(H\) can be written using (8), (15) simplified by neglecting \(\rho_g\), and (18) as:

\[
H = \int_{P_\infty}^{p} \frac{1 + f(p)}{\rho_l} dp.
\] (19)

Carrying out the integration, we have the value of \(H\) required to integrate (7)

\[
H = \frac{P_b - P_{\infty}}{\rho_l} + \left( A + D \right) \frac{k}{\rho_l(k - 1)} p_0^{\frac{k}{k-1}} \left( P_b^k - P_{\infty}^k \right) - \frac{D}{2 p_0 \rho_l} \left( P_b^2 - P_{\infty}^2 \right).
\] (20)

The value of \(c_b\) also required for the integration of (7) is simply obtained using equations (16) and (17).

The results of the Gilmore-based model described above have been verified to be accurate by comparison against the results of a finite element multi-component compressible flow solver (GEMINI) developed by NSWCDIH for the solution of underwater explosion bubbles for the case of a spherical bubble expanding and collapsing in a continuum bubbly mixture [11,13].
Obviously, the above presented continuum model is not expected to capture the full physics of a spherical bubble expanding/collapsing in a bubbly medium since the model does not account for viscosity, and more importantly for the local individual and collective behavior of the bubbles in the medium surrounding the primary bubbles. Thus, the model is not expected to account for any oscillatory behavior due to distinct response times of the bubbles that form the bubbly medium surrounding the primary bubble in consideration. The objective, however, is to compare the results of this simple model (similar to many existing conventional two-phase flow approaches) against experimental results. This is expected to provide an insight on the extent to which a discrete bubbly mixture can be modeled as a continuum medium.

IV. RESULTS AND DISCUSSION

d. Visualizations

The spark tests presented here consider spark-generated bubbles in the absence of surrounding microbubbles (i.e. liquid only tests) as compared to those conducted in the presence of a bubbly medium generated by electrolysis. Figure 5 shows a time sequence from a high-speed movie of the primary bubble dynamics growing and collapsing in a supposed “pure” liquid, while Figure 6 shows a similar sequence of the bubble generated in the bubbly medium. Both figures also show the response of the surrounding bubbles to the resulting pressure and flow field. Figure 5 is supposed to illustrate the case of a bubble generated in pure water liquid, as the water was degassed for a long time and shaken extensively to remove any remaining gas bubbles. However, as can be seen in the selected sequence, some isolated nuclei remained attached to the front window through which the movies is taken. These unwanted bubbles are actually useful in highlighting the pressure at their location and are seen to respond to the primary bubble pressure field and to show some secondary bubble dynamics. As the primary bubble wall accelerates away from the spark gap a very strong pressure rise occurs and is seen to strongly affect the unwanted bubbles at the wall. (The field pressures can be seen in Figure 12 and Figure 13 and are discussed further later). They collapse and rebound between the first and the second frame and again between the second and third frame. A couple of them are even seen to collapse and merge on the left and below the electrodes. The initial pressure spike is followed by a long period of pressure drop (as the primary bubble goes through its growth phase). This forces the small bubbles at the wall to grow and interact. As the primary bubble collapses, a second pressure spike is emitted which forces the small bubbles to collapse at their turn and form reentrant jets against each other.
Figure 6 illustrate the interaction between the spark bubble and the two-phase medium. Here again, two different behaviors can be seen a) during the high acceleration phases of the bubbles (initial growth and collapse) where the two-phase medium is in compression and b) during the low pressure phase, where the liquid around the bubble is in expansion. This is illustrated mainly in the 2nd picture where a region of about 0.5 times the bubble around the bubble has very low void fraction, and in the previous to last picture where a much larger portion is affected, while the region very close to the main bubble sees streaks due to very large translation speeds of the surrounding microbubbles. During Phase a) the void fraction around the main bubble is seen to go down, while it increases more noticeably during the lengthier Phase b). This effect is the strongest at the times where the primary bubble is close to its maximum radius. In addition to the above there a tendency to create stratification of the microbubbles in radial rings, which is unfortunately easier to observe in the high speed movies than in the still pictures shown here.

### e. Bubble Dynamics Analysis

Quantitative analysis of the movies is shown in Figure 7 and Figure 8. These present the primary bubble equivalent spherical radii vs. time in the pure liquid (i.e., no bubble injection) and in the bubbly medium as described above. The data was extracted through image analysis techniques of the movie images [14]. The bubble outlines were extracted from the recorded high-speed movies of the spark tests. Then, the volume vs. time of the bubble was computed as the volume of a body of revolution with an axis perpendicular to the visualization wall. Finally, the bubble equivalent radii were computed by using the bubble volume.

Three repeats for each case are shown. Variations between the repeats are attributed to change in the electrodes condition between discharges and to the known non perfect repeatability in the spark discharge mechanics [16]. In a parametric study these variations can be accounted when the data is represented in a normalized fashion relative to the maximum bubble size achieved [19,20]. This procedure is however not helpful if we want to see the effect of the presence of the two-phase microbubble medium on the dynamics of the primary bubble as normalization smears out the sought effect.
Figure 7 presents the results before any normalization. Despite repeat issues, some clear trends can be seen. Scatter in the repeats is larger when microbubbles are present and this is understandable as the conductivity of the two-phase medium is much harder to conserve between two runs in this case. In addition potential for microbubble contamination of the electrodes increase significantly in these cases. Repeatability for the ‘pure’ water tests seems to be much better. Another salient result, which is reproduced by the numerical simulations, is that the presence of
the bubbly flow significantly affects the dynamics and the maximum bubble sizes, $R_{\text{max}}$, achieved by the primary bubble. The bubbly liquid, formed of smaller microbubbles, reduces the maximum size and the period of the primary bubble in an observable manner. The bubble size is reduced probably due to transfer of energy from the primary bubble to the surrounding bubbles in the case of a two-phase medium. Reduction of the period could be due to the same effect, i.e. could be a result of the reduction of $R_{\text{max}}$.

Actually, analysis of the non-dimensional curves in Figure 8 reveals that this is not the case. In this figure the equivalent radii is normalized with the achieved maximum radius of the bubble, $R_{\text{max}}$, and the time with Rayleigh-time, $t_{\text{rayleigh}}$ based on $R_{\text{max}}$ and the experimental ambient pressure in the tank as defined in (1). The figure clearly shows that the time to reach the same $R_{\text{max}}$ from the initiation of the primary bubbles is not modified by the presence of the surrounding bubbles. However, the time to collapse from $R_{\text{max}}$ is noticeably increased. This indicates the damping effect that the two-phase medium has on the bubble collapse.

The normalized bubble collapse rate is seen in Figure 8 to decrease in the bubbly medium, which means the actual dimensional rate also decreases since the reference velocity, $\sqrt{(P_{\text{amb}} - p_v) / \rho}$, does not practically change between the two cases. This, along with the reduced $R_{\text{max}}$, is what primarily affects the violence of the primary bubble collapse and in turn reduces the magnitude of the spike in the field pressure observed during primary bubble collapse as observed in Figure 12 and Figure 13.
Figure 7. Comparison of the evolution of the primary bubble radius vs. time for spark-generated bubbles in ‘pure’ water and in a bubbly medium.

Figure 8. Comparison of the evolution of the normalized primary bubble radius vs. normalized time for spark-generated bubbles in ‘pure’ water and in a bubbly medium.
In order to examine if these effects can be recovered through the simple continuum model approach described above, we use the experimental conditions to integrate the Gilmore equation combined with the bubbly medium equation of state presented earlier. To exercise the code, Equation (7) needs appropriate initial conditions to be integrated. These should reflect the correct energy imparted to the bubble through the spark discharge. To do so we use, as is conventional [16,20], the spherical dynamics of the bubble in an infinite domain and in the pure liquid to obtain what is necessary. One quantity easy to measure is the maximum bubble radius. However, the initial bubble radius, \( R_0 \), and the initial gas pressure in the bubble, \( P_{g0} \), are unknown. \( R_0 \) has an upper limit, which is given by the bubble radius in the first frame of the high speed movie. To estimate \( R_0 \) and \( P_{g0} \) we conduct an iterative optimization procedure to match the numerical curve \( R(t) \) with the corresponding experimentally measured curve, while imposing \( \dot{R}_0 = 0 \). These same values of \( R_0 \) and \( P_{g0} \) are then used for all other simulations.

Figure 9 shows comparisons between the analytical/numerical model and the experiments. The figure shows the evolution of the primary bubble radius versus time in pure water and in a bubbly medium with an initial void-fraction, \( \alpha_0 = 1\% \). In the ‘pure liquid’ the correspondence between the iteratively determined numerical solution and the experiments (note that the curve is an average curve of three spark-generated experiments) is obviously quite good during most of the bubble dynamics. However, we can see some discrepancies at the end of the curves. The numerical solution, which obviously assumes perfect spherical symmetry, predicts a much smaller value of \( R_{min} \) than what is observed experimentally. In the experiments gravity effects (and presence of unwanted microbubbles) become important at the end of the collapse and terminate the collapse earlier than the predictions.

In presence of the two-phase medium, the numerical solution using the same values of \( R_0 \) and \( P_{g0} \) and \( \dot{R}_0 = 0 \) appears to also provide a good match with the experimental data obtained by averaging the observations from three spark-generated bubble tests. The maximum bubble radii appear to match between experiments and the numerical solution. However, the period appears to be about 4% smaller in the modeling than in the experiments. This discrepancy seems to be systematic as it shows up both during the bubble growth and collapse. This indicate
some mismatch in the value of \( (P_{\text{amb}} - p_v) \) between the experiments and the simulations, which requires further investigation.

Figure 10 shows non-dimensional representations of the plots in Figure 9. Here again, it is clear from the figures that the simple analytical model captures quite well the observed overall characteristics: a reduction in the maximum bubble radius and a reduction in the time period of bubble oscillations when the medium is bubbly. The correspondence remains very good for the normalized curves where the presence of a void fraction is seen to increase (not decrease) the normalized period. Discrepancies at the end of the collapse indicate, however, that a more accurate modeling is required to capture well the collapse. This reflects itself in the much stronger damping observed experimentally than numerically.

![Figure 9. Evolution of the primary bubble radius vs. time – Comparison of spark-generated bubble results and the analytical model. Experimental data are average of the 3 experiments.](image)
Figure 10. Evolution of the primary bubble normalized radius vs. normalized time – Comparison of spark-generated bubble results and the analytical model. Experimental data are average of the 3 experiments.

\[ \frac{R}{R_{\text{max}}} \]

\[ \frac{t}{t_{\text{Rayleigh}}} \]

**g. Field Void Fraction Variations**

The void fraction in the field can give further information of the dynamics of a spherical bubble in a two-phase medium. We select here a point located at a radial distance of 1.71 cm from the spark electrodes for the case \( \alpha_0 = 1\% \) corresponding to previous results in Figure 6, Figure 9, and Figure 10. Shown in Figure 11 is the void fraction measured. Also shown in the figure are the void fractions versus time computed at the same location using the present analytical/numerical method. Very significant differences can be seen between the two results: while the experimental void fraction measurements show very strong fluctuations, the analytical computations capture only smoothed “time averaged” or frequency filtered values. We believe that this is due to the fact that the experiments capture the actual dynamics of individual microbubbles, which, as described earlier, oscillate, grow and collapse based on their initial radii, in response to the local pressure they encounter. On the other hand the present analytical/numerical model does not directly account for the discrete microbubbles dynamics. Instead, it lumps the micro-bubble effects into a density variation though the averaged void fraction of a continuum bubbly medium. This highlights the need for a model, which is able to both describe the macro-scale as a continuum and which accounts at
the micro-scale for the individual bubble dynamics. Such a model is being developed in parallel to this study [13] and has already shown promise to capture behavior as shown in Figure 11.

![Graph showing void fractions](image1)

*Figure 11. Comparison of the void fractions computed from experiments and the analytical/numerical model at 1.71 cm from the electrodes. Also shown are pictures from a high speed movie of the instantaneous bubble size distribution.*

**h. Field Pressures**

An important aspect of the dynamics of a bubble in a two-phase flow is related to the pressures generated by the dynamics, bubble generation, oscillation, growth, and collapse. We examine here the pressure measured by a transducer located flush in the wall (to minimize interference with the flow field) at a radial distance of 11 cm from the electrodes/spark center. The measured pressures in Figure 12 can be seen to be very different between the one
obtained with the spark generated in the pure liquid and that obtained with the $\alpha=1\%$ two-phase medium. The figure clearly shows in the pure liquid, the initial pressure peak generated by the spark followed by a significant pressure drop and several acoustic reflections and pressure oscillations. This phase is followed by a long duration low level pressure below the ambient static pressure and finally by a very large pressure peak generated during the final bubble collapse. This description is significantly modified when a two-phase medium exists around the primary spark-generated bubble. Following the spark peak, the pressure drop and pressure oscillations is very much attenuated by the presence of the two-phase flow and the $p(t)$ curve appears much smoother in the same time region. Later on, but earlier than for the bubble in the pure liquid-- due to the reduction in the bubble period, as seen earlier from the bubble dynamics -- the bubble collapse results in a much weaker pressure peak as seen at $t\sim 0.0085s$. This is due to both the weakening of the bubble collapse and the attenuation of the pressures in the bubbly medium.

Figure 13 addresses the same issues analytically/numerically. It is clear from the figure and from comparison between Figure 13 and Figure 12 that the present simple numerical model captures the key low frequency features of the dynamics. Both amplitude and timing of the pressure peaks and their relative values are captured. However, more fine high frequency details are beyond the capabilities of the model and again a combined macroscopic – microscopic model is needed to capture all features.
Figure 12. Pressure versus time recorded by a transducer located at a distance of 11 cm from the bubble center.

Figure 13. Pressure versus time computed by the present analytical/numerical model at a distance of 11 cm from the bubble center. Initial conditions are $R_0 = 0.0033 \text{ m}$ and $P_{g0} = 1,512,942 \text{ Pa}$. 
**CONCLUSIONS**

In order to better understand the effect of a bubbly medium on the dynamics of a primary bubble, experiments were conducted with a spark generated hemispherical bubble near a rigid transparent wall, in both pure liquid and in bubbly medium. The evolutions of the primary bubble radius in pure and a bubbly medium were compared. It was shown that the presence of the bubbly medium dampens the growth of the primary bubble and slows down the rate of bubble collapse.

The experimental data was used to compare against an analytical model developed based on Gilmore’s Equation. The model was also able to capture the same trend as seen in the experiments.

Finally, with preliminary data both from the experiments and the analytical model, the mitigation effects of a bubbly medium were qualitatively confirmed.

The study is now being pursued with more extensive experimental and numerical conditions using spark bubbles of different sizes, ambient pressures, and void fractions and the results will be used in the development and validation of an Eulerian-Lagrangian code which takes into account the dynamics of the bubbles in the two-phase medium.

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