ABSTRACT
Cavitation is a problem of interaction between nuclei and local pressure field variations including turbulent oscillations and large scale pressure variations. Various types of behaviors fundamentally depend on the relative sizes of the nuclei and the length scales of the pressure variations as well as the relative importance of the bubble natural period of oscillation and the characteristic time of the field pressure variations. Ignoring this observation and basing cavitation inception predictions on pressure coefficients of the flow of the pure liquid, without account for bubble dynamics could result in significant errors in predictions. We present here a practical method using a multi-bubble Surface Averaged Pressure (DF-Multi-SAP) to simulate cavitation inception and scaling, and connect this with more precise 3D simulations.

INTRODUCTION
Cavitation and bubble dynamics have been the subject of extensive research since the early works of Besant in 1859 [1] and Lord Rayleigh in 1917 [2]. Thousands of papers and articles and several books [e.g., 3-10] have been devoted to the subject. Various aspects of the bubble dynamics have been considered at length under various assumptions and each contribution included one or several physical phenomena such as inertia, interface dynamics, gas diffusion, heat transfer, bubble deformation, bubble-bubble interaction, electrical charge effects, magnetic field effects, ...etc. Unfortunately, very little of the resulting knowledge has succeeded in crossing from the fundamental ‘research world’ to the ‘applications world’, and it is uncommon to see bubble dynamics analysis made or bubble dynamics computations conducted for cavitation avoidance by the hydrodynamics marine designer community, such as propeller designers. This is due in part to the failure of the scientific community to frame the advances made in a format usable by the design community, but much more importantly to the perceived impracticality of using the methods developed with the existing design resources. This has made the use of bubble dynamics seem inconceivable but by experts.

Recently, however, there have been tremendous advances in available computing resources placed at the reach of non-experts. Personal computers with phenomenal speed, memory, and storage size, when compared to what existed a decade ago, are now in the hands of most engineers at a small fraction of the cost of an entry computer a decade ago. This computer ‘revolution’ has definitely affected the operating procedures of the designers. For instance, while a few years ago, use of CFD viscous solvers by designers was out of reach and only very simplified codes were used to design and model rotating machinery, it is now common to use repeatedly in-house or commercial Navier Stokes solver CFD codes to seek better solutions [11-13]. The challenge is thus presently for the cavitation community to bring its techniques to par with the single phase CFD progress. It is this challenge that is been undertaken here and to which we wish to significantly contribute.

In this paper, we discuss first the various definitions of cavitation because of their significant implications on modelling and then describe the analytical and numerical tools that have become available. We will try to convey the need to include the presence of nuclei and nuclei dynamics in the predictive tools for advanced designs. Some of these tools are at the reach of all users and should be adopted by the design community in conjunction with the CFD tools presently used for advanced design.

Definition(s) of Cavitation
Liquid phase only: Engineering definition
In the phase diagram of a substance the curve which separates the liquid phase from the
vapour phase defines the liquid vapour pressure values at different temperatures. Any process that raises the temperature or reduces the pressure will result in a phase change from liquid to vapour. Conventionally, boiling is defined as the phase change resulting from raising the temperature at ambient pressure, while cavitation is the process inducing phase change at ambient temperature through a pressure drop. This has provided the following traditional cavitation engineering definition: “a liquid flow experiences cavitation if the local pressure drops below the liquid vapour pressure, $P_v$.”

One root of the technology transfer problem discussed above stems from this accepted engineering definition of cavitation. Even though this definition has allowed significant progress in practical cavitation studies and design work, it is responsible for a lack of further advance of the technology, since it has been used at many decision points to ignore bubble dynamics effects. Indeed this definition assumes that the process occurs in the regime where heat transfer is negligible and where a large free surface is present. This over-simplification serves the purpose in most engineering cases but could lead to erroneous conclusions if used to explain or model new complex problem areas. The dangerous implication of this definition is that understanding of the liquid one-phase flow only is sufficient to predict and therefore avoid cavitation.

We discuss in the following more advanced definitions, which can help us to better understand the scaling of the cavitating results between laboratory and full scale. They should also help cavitation test results comparison between different testing facilities, and enable making more accurate cavitation predictions.

**Presence of cavitation nuclei**

The above definition of cavitation inception is only true in static conditions when the liquid is in contact with its vapour through the presence of a large free surface. For the more common condition of a liquid in a flow, or in a rotating machine, liquid vaporization can only occur through the presence of “micro free surfaces” or microbubbles, also called “cavitation nuclei”. Indeed, a pure liquid free of nuclei can sustain very large tensions, measured in the hundreds of atmospheres, before a cavity can be generated through separation of the liquid molecules [3]. In fact researchers agree that cavitation initiates at weak spots of the liquid or nuclei. These are very small microscopic bubbles or particles with gas trapped in crevices in suspension in the liquid. Several techniques have been used to measure these nuclei distributions both in the ocean and in laboratory cavitation channels. These include Coulter counter, holography, light scattering methods, cavitation susceptibility meters, and acoustic methods [15-21]. Figure 1 shows typical nuclei size distribution curves in cavitation water tunnels and in the ocean [14, 22, 23]. The figure shows the number density distribution, $n$, in $m^{-4}$, as a function of the bubble size, $R$. $n(R_a)$ is the number of nuclei bubbles in the range $R_a$ to $R_a+dR$. Distributions of the shape $n(R) \approx R_4$ are usually reported.

Therefore, any fundamental analysis of cavitation inception has to start from the observation that, any real liquid contains nuclei which when subjected to variations in the local ambient pressure will respond dynamically by oscillating and eventually growing explosively (i.e. cavitating).

![Figure 1. Nuclei Size distribution as measured in the ocean and in the laboratory (from [22].)](image-url)

Cavitation inception in fact appears under several forms, such as:

a. Explosive growth of individual travelling bubbles,

b. Sudden appearance of transient cavities or “flashes” on boundaries,

c. Sudden appearance of attached partial cavities, or sheet cavities,
d. Appearance, growth, and collapse of bubble clouds, behind attached cavities or a vibrating surface.

e. Sudden appearance of cavitating rotating filaments, or vortex cavitation.

Upon further analysis, all these forms can be related initially to the explosive growth of pre-existing cavities or nuclei in the liquid when subjected to pressure drops generated by various forms of local pressure disturbances\(^1\). These are either imposed pressure variations, uniform pressure drops due to local liquid accelerations, or strongly non-uniform pressure fields due to streamwise or transverse large vortical structures. The presence of nuclei or weak spots in the liquid is therefore essential for cavitation inception to occur when the local pressure in the liquid drops below some critical value, \(P_c\), which we address next.

**Bubble Static Equilibrium**

A first correction to the common engineering definition of cavitation inception is based on consideration of the static equilibrium of a bubble nucleus. The nucleus is assumed to be spherical and to contain non-condensable gas of partial pressure, \(P_g\), and vapor of the liquid of partial pressure, \(P_v\). Therefore, at the bubble surface, the balance between the internal pressure, the liquid pressure, and surface tension can be written:

\[
P_L = P_v + P_g - \frac{2\gamma}{R},
\]  

where \(P_L\) is the pressure in the liquid, \(\gamma\) is the surface tension parameter, and \(R\) is the radius of the bubble.

If the liquid ambient pressure changes very slowly, the bubble radius will change accordingly to adapt to the new balance. This is accompanied with a modification of the pressure inside the bubble. The vaporization of the liquid at the bubble-liquid interface occurs very fast relative to the time scale of the bubble dynamics, so that the liquid and the vapor can be considered in equilibrium at every instant, and the partial pressure of the vapor in the bubble remains constant. On the other hand, gas diffusion occurs at a much longer time scale, so that the amount of gas inside the bubble remains constant\(^2\). This results in a gas partial pressure which varies with the bubble volume. For quasi-steady equilibrium, \(P_c\) as considered in this section, the gas follows an isothermal compression law, and is related to the initial or reference values, \(P_{go}\), \(R_0\), and to the new bubble radius \(R\) through:

\[
P_g = P_{go} \left( \frac{R_0}{R} \right)^3.
\]  

The balance of pressures at the bubble wall becomes:

\[
P_L(R) = P_v + P_g \left( \frac{R_0}{R} \right)^3 - \frac{2\gamma}{R},
\]  

where the notation \(P_l(R)\) is meant to associate the liquid pressure, \(P_L\) to the bubble radius, \(R\). An understanding of the bubble static equilibrium can be obtained by considering the curve; \(P_l(R)\). As illustrated in Figure 2, this curve has a minimum below which there is no equilibrium bubble radius. Only the left side branch of the curve corresponds to a stable equilibrium.

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\(^1\) This could be followed by extreme bubble deformation and merger to result in the various cavitation forms.

\(^2\) More generally, both gas diffusion and vaporization can be modeled and taken into account.
If the pressure in the flow field drops below the “critical pressure” an explosive bubble growth, i.e. cavitation, is provoked. This provides an improved definition for cavitation inception: “a liquid flow experiences cavitation if the local pressure drops below the critical pressure, \( P_c \).” The reason such a definition has not been adopted is that the critical pressure is not a liquid only definition and a different value is obtained for each nucleus size. To use it, one requires knowledge of the nuclei size distribution in the liquid (which is ultimately needed for any serious scale up study of cavitation.) Expression (4) illustrates the fact that the critical pressures are always lower than the vapor pressure. \( P_c \) is close to \( P_v \) only for very large initial nuclei sizes. This probably explains why such a criterion has been ignored by the practitioners, the reasoning being that using \( P_v \) is on the safe side. This reasoning, however, results in large margins of safety. In addition, this cannot be used to scale up experimental small scale tests, since cavitation would occur when bubbles actually grow explosively in the laboratory experiments and not when \( p = P_v \), but the scaling would assume \( p = P_c \).

Cavitation inception cannot be defined accurately independent of the liquid bubble population (sometimes characterized by liquid “strength” [24]) and independent of the means of cavitation detection. The cavitation inception is in fact a complex dynamic interaction between the nuclei and their surrounding pressure and velocity fields; interaction that can be different between small and large scales. In addition, the experimental means to detect and decide cavitation inception (practical threshold used by the experimentalists) will affect the results and could be different between a laboratory experiment and full scale.

**Dynamical Effects**

When the pressure variations to which the bubble is subjected are not slow compared to the bubble response time, the nuclei cannot instantaneously adapt to the new pressure, inertia effects become important, and thus one needs to consider the bubble dynamics equation. This is the case for nuclei travelling through a rotating machinery. The nuclei /bubbles then act as resonators excited by the flow field temporal and spatial variations. In the case of a vortical flow field the strong spatial pressure gradients (in addition to the temporal) strongly couple with the actual bubble motion (i.e. position vs time) to result in a driving force that depends on the resonator reaction. This makes such a case much more complex than what occurs for a travelling bubble about a foil where, relatively speaking, the position of the bubble is less coupled to its dynamics.

The flow field pressure fluctuations have various time scales: e.g. relatively long for travelling cavitation bubbles over a blade or captured in a vortical region flow, or very short for cavitation in turbulent strongly sheared flow regions. The amplitudes of these fluctuations and the relationship between the various characteristic times determine the potential for cavitation inception.

**Spherical Bubble Dynamics**

The first improvement to the static equilibrium analysis of a bubble nucleus is to consider the nucleus dynamics when it is assumed to conserve a spherical shape during its motion. This has been extensively studied following the original works of Rayleigh [1] and Plesset [25]. For instance, if we limit the phenomena to be modelled to inertia, small compressibility of the liquid, compressibility of the bubble content, we obtain the Gilmore [26] differential equation for the bubble radius \( R(t) \). We modified this equation to account for a slip velocity between the bubble and the host liquid, and for the non-uniform pressure field along the bubble surface [27]. The resulting Surface-Averaged Pressure (SAP) equation applied to Gilmore’s equation [27-28] becomes:

\[
(1-\frac{\dot{R}}{c})\dot{R}R + \frac{3}{2}(1-\frac{\dot{R}}{3c})\dot{R} = \frac{1}{\rho}(1+\frac{\dot{R}}{c} + \frac{R}{c} \frac{d}{dt}) \left[ P_v + p_b - p_{\text{encounter}} - \frac{2\sigma}{R} - 4\mu \frac{\dot{R}}{R} \right] + \frac{(u - u_b)^2}{4},
\]

where \( c \) is the sound speed, \( \mu \) is the liquid viscosity, \( u \) is the liquid convection velocity and \( u_b \) is the bubble travel velocity.

Equation (5) degenerates to the classical Rayleigh-Plesset [10] equation for negligible compressibility effects. If in addition, gas diffusion effects are neglected and a polytropic law of gas compression is assumed, the resulting modified SAP equation becomes:
\[ R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{1}{\rho} \left( p_\lambda + p_\varnothing \left( \frac{R_\lambda}{R} \right)^{3\gamma} - P_{\text{encounter}} \right) + \]
\[ - \frac{1}{\rho} \left( \frac{2\gamma}{R} + \frac{4\mu \dot{R}}{R} \right) + \frac{\left( u - u_\lambda \right)^2}{4}. \]

where \( k \) is the polytropic compression law constant.

In the Surface-Averaged Pressure (SAP) bubble dynamics equation, we have accounted for a slip velocity between the bubble and the host liquid, and for a non-uniform pressure field along the bubble surface. In this SAP method the definition of \( P_{\text{encounter}} \) as the average of the liquid pressures over the bubble surface results in a major improvement over the classical spherical bubble model which uses the pressure at the bubble center in its absence [27-29]. For instance, a bubble does not always continuously grow once it is captured by a vortex. Instead, it is subjected to an increase in the average pressure once it grows and this leads to a more realistic bubble dynamics. In general, the gas pressure, \( p_g \), is obtained from the solution of the gas diffusion problem and the assumption that the gas is an ideal gas [30].

The bubble trajectory is obtained using the following motion equation [32]
\[
\frac{d\textbf{u}_b}{dt} = \frac{3}{\rho} \nabla P + \frac{3}{4} C_D \left( \textbf{u} - \textbf{u}_b \right) \times \textbf{u} - u_b + C_L \left( \textbf{u} - \textbf{u}_b \right) + \frac{3}{R} \left( \textbf{u} - \textbf{u}_b \right) \dot{R},
\]
where the drag coefficient \( C_D \) is given by an empirical equation such as that of Haberman and Morton [31]:
\[
C_D = \frac{24}{R_{ch}} \left( 1 + 0.197 R_{ch}^{0.63} + 2.6 \times 10^{-4} R_{ch}^{3.8} \right);
\]
\[
R_{ch} = \frac{2 \rho \mu}{R \left| \textbf{u} - \textbf{u}_b \right|}.
\]

Non-spherical Bubble Dynamics: Axisymmetry

Spherical bubble models, as briefly described above, can be efficient tools for studying cavitation inception, scaling, bubble entrainment, and cavitation noise. They can become more powerful if they are provided with further “intelligence” based on more precise non-spherical models which account for bubble behavior near boundaries, in pressure gradients, and in high shear regions, resulting in bubble deformation, elongation, splitting, coalescence, and non-spherical sound generation.

One such refinement, important for propulsor studies, consists of considering the case of bubbles captured on a vortex axis. The bubble then elongates along the axis and may split into two or more sub-bubbles, and/or form jets on the axis. In order to investigate this behavior the commercial boundary element method axisymmetric bubble dynamics code 2DYNAFS© [33-38] was exercised and was able to simulate bubble dynamics through reentrant jet formation, jet break through, and bubble splitting. The code can handle as input vortex flow fields obtained from CFD viscous computations or from experimental measurements.

![Figure 3: Illustration of the acoustic pressure emitted by a bubble in a vortex field as a function of the cavitation number. Note that the bubble behavior near and above the cavitation inception is quasi-spherical [35].](image)

By simulating the dynamics behavior of a bubble captured on a vortex axis under a significant number of conditions using the 2DYNAFS©, the followings conclusions illustrated in Figure 3 were found [35,41]:

- If the bubble is captured by the vortex far upstream from the minimum pressure, it remains spherical while oscillating at its natural frequency.
- When the bubble reaches the axis just upstream of the minimum pressure, it develops an axial jet on its downstream side which shoots through the bubble moving in the upstream direction. Even at this stage, the spherical model provides a very good approximation because the bubble is more or less spherical until a thin jet develops on the axis.
• The bubble behavior becomes highly nonspherical once it passes the minimum pressure location. It elongates significantly and can reach a length to radius ratio that can exceed 10. The bubble then splits into two or more daughter bubbles emitting a strong pressure spike followed later by other strong pressure signals when daughter bubbles collapse. Two axial jets originating from the split and a strong pressure signal during the formation of the jets are observed.

Experimental Verification

This behavior supports the hypothesis that the noise at the inception of the vortex cavitation may originate from bubble splitting and/or the jets formed after the splitting. This is an important conclusion that has been preliminarily confirmed experimentally [35,38].

Three types of tests were conducted and are still on-going: spark generated bubbles, laser generated bubbles, and electrolysis bubbles injected in vortex lines. Figure 4 shows high speed photography and acoustic signals of bubble splitting between two rigid walls. A small but distinct pressure spikes is formed at splitting followed but a more significant spike during the collapse of the sub-bubbles. The second set of experiments was conducted in a vortex tube, where bubbles generated by electrolysis were injected and observed once captured by the vortex line. Figure 5 shows the elongated bubble dynamics and the corresponding signals measured by a hydrophone [38]. The third set of experiments was conducted at the University of Michigan [39] using laser induced bubbles (in the vortex and far upstream) and the flow field of a tip vortex behind a foil. Comparisons between the observations and the 2DYNAFS© simulations showed very good correspondence as shown in Figure 6.

![Figure 4. High speed photos of spark-generated bubble collapsing between two solid walls, and resulting acoustic signal indicating peak signal at splitting and subsequent sub-bubbles collapse [35]](image)

![Figure 5. High speed photos of an electrolysis bubble captured in a line vortex, and resulting acoustic signal indicating peak signals at splitting and collapse [40].](image)

![Figure 6. Bubble behaviour in a vortex flow field: \( R_e = 4.51 \text{mm} \), \( \Gamma = 0.2123 \text{ m}^3/\text{s} \), \( \sigma = 1.72 \); \( U_\infty = 10 \text{ m/s} \); \( R_0 = 750 \mu \text{m} \). Three-dimensional view of the bubble just before splitting predicted by 2DYNAFS© and observed at University of Michigan using laser-induced bubbles at the center of the vortex [39].](image)

**Bubble splitting criteria**

A large series of computer simulations of axisymmetric bubbles captured in a vortex indicated some definite trends, which can be used in a predictive model [34-38]:
An explosively growing bubble splits into two sub-bubbles after it reaches its maximum volume, (equivalent radius, $R_{\text{max}}$) and then drops to 0.95 $R_{\text{max}}$.

- The two resulting sub-bubbles have the following equivalent radii: 0.90 $R_{\text{max}}$ and 0.52 $R_{\text{max}}$.
- The locations of the two sub-bubbles after the splitting are at -0.95 $R_{\text{max}}$ and 4.18 $R_{\text{max}}$.
- The pressure generated by the subsequent formation of reentrant jets in the sub-bubbles can be approximated by a function of $\sigma$ [41]:

Since the noise associated with the jet formation appears to be much higher than the pressure signal from the collapse of a spherical bubble, it is desirable to include the splitting and the associated jet noise in simulations with multiple bubble nuclei.

**Fully Non-spherical Bubble Dynamics**

In order to study the full 3D interaction between a bubble and a complex flow field, two methods were developed. The first, using the commercial boundary element code, 3DYNAFS© [42], enables study of full bubble deformations during capture but neglects the effects that the bubble may have on the underlying flow field. The second method accounts for the full two-way bubble/flow field interaction, and considers viscous interaction. This model is embedded in an Unsteady Reynolds-Averaged Navier-Stokes code, DF-UNCLE³, with appropriate free surface boundary conditions and a moving Chimera grid scheme [28,42]. This full two-way interaction non-spherical bubble dynamics model has been successfully validated in simple cases by comparing the results with reference results obtained from the Rayleigh-Plesset equation and 3DYNAFS for bubble dynamics in an infinite medium both with and without gravity [43].

As an illustration Figure 7 shows results of a bubble interacting with the tip vortex of an elliptical foil. The bubble elongates once it is captured, and depending on the cavitation number, either forms a reentrant jet directed upstream or splits into two sub-bubbles. When two-way interaction is taken into account further smoothing of the bubble surface is exerted by viscosity resulting in a more distorted but overall more rounded bubble. Figure 8 illustrates the various stages of the interaction between a bubble and a tip vortex flow.

**Validation of the SAP model**

In order to evaluate the various models, we combined the SAP spherical model and the two-way interaction non-spherical bubble dynamics model to predict tip vortex cavitation inception for a tip vortex flow generated by a finite-span elliptic hydrofoil [28].

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³ *DF UNCLE is a DYNAFLOW modified version of UNCLE developed by Mississippi State University*
model in which unsteady viscous computations included modification of the flow field by the presence of the bubbles.

- Differences between the one-way and two interaction models exist but are not major.
- Using the Surface Averaged Pressure (SAP) scheme significantly improves the prediction of bubble volume variations and cavitation inception. SAP appears to offer a very good approximation of the full two-way interaction model.

Comparisons between the various models of the resulting bubble dynamics history, and of the cavitation inception values obtained from many tested conditions, reveal the following conclusions, illustrated in Figures 9 and 10:

- The bubble volume variations obtained from the full two-way interaction model deviate significantly from the classical spherical model due to the interaction between the bubble and the vortex flow field.

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duration of the simulation and the characteristic velocity in the release area as illustrated in Figure 11. The liquid considered has a known nuclei size density distribution function, \( n(R) \), which can be obtained from experimental measurements [16-21] and can be expressed as a discrete distribution of \( M \) selected nuclei sizes. Thus, the total void fraction, \( \alpha \), in the liquid can be obtained by

\[
\alpha = \sum_{i=1}^{M} N_i \frac{4\pi R_i^3}{3},
\]

where \( N_i \) is the discrete number of nuclei of radius \( R_i \) used in the computations. The position and thus timing of nuclei released in the flow field are obtained using random distribution functions, always ensuring that the local and overall void fraction satisfy the nuclei size distribution function.

**Vortex/Vortex Interaction and Inception**

Recent experiments on ducted propellers [44] have shown a cavitation inception value and a cavitation inception location which were very different than those predicted by several state of the art CFD RANS codes. The most disturbing conclusion made from this comparison was that cavitation inception does not occur in the minimum pressure region, which would contradict our understanding of cavitation inception as the explosive growth of nuclei in low pressure regions.

One hypothesis for explaining this doubtful conclusion was that in both the experiments and the simulations unsteady effects were not accounted for, with the RANS solutions smearing out the computed fluctuations and the experimental measurement techniques filtering them through time averaging. The reason this effect was enhanced in the concerned experiment is that the flow field was inherently unsteady and thus significantly affected bubble dynamics in a complex fashion. Indeed, in the considered ducted propeller there is strong interaction between a tip-leakage vortex and a trailing-edge vortex as illustrated in Figure 12 at a low cavitation number, showing cavitation development and interaction between the two structures. This evolving vorticity may cause early cavitation wherever the two vortices strongly interact. We therefore set out to analyze whether unsteady vortex/vortex interactions affect bubble dynamics in a way as to explain the above observations.

To address this issue we exercised the methods described above to study the effect of vortex/vortex interaction on bubble dynamics and cavitation noise [45,46]. The liquid phase flow was solved by direct numerical simulation of the Navier-Stokes equations and was coupled with the SAP spherical bubble dynamics model to track the evolution of the bubbles at each time step.

**Canonical problem**

A canonical problem was considered first. Bubble dynamics in the flow field of two-unequal co-rotating vortices with different configurations was considered and resulted in the following conclusions [45].

- A stronger interaction between the two vortices was observed when the strengths of the two vortices were closer.
- The minimum pressure value and location is strongly affected by the two-vortices interaction. It could occur at, before, or after the two vortices have completely merged depending on the relative strength of the two vortices (see Figure 13).
- The pressure reaches its minimum when the vorticity of the weaker vortex is spread and sucked into the stronger vortex. This also results in an acceleration of the flow and leads to a maximum streamwise velocity at the vortex center.
The shape, size and location of study of the “window of opportunity”, i.e. area in the inlet to the computational domain from which emitted nuclei are captured in the vortices, are highly dependent on the relative strength of the two vortices and on the nuclei sizes. A large size of “window of opportunity” was found for the stronger interaction case and for larger nuclei.

The unsteady flow resulting from the interaction of the two vortices may result in some nuclei initially starting to be entrapped by one vortex to be ejected by the other during the merging process (see Figure 13).

**Propulsor study**

The same approach as discussed above was applied to the David Taylor Propeller 5206 [44] shown in Figure 12. Three RANS codes have been used by three groups to simulate numerically cavitation inception on this propulsor [46-49]. All three codes followed the simple engineering criterion for cavitation inception, \( \sigma_i = -c_{p_{min}} \), and gave close inception values: \( 6.5 < \sigma < 8 \), at a location 0.1 chord length downstream of the trailing edge of the blade. The experiments however, showed a much higher inception value, \( \sigma = 11 \), and, more disturbing, at a location much further downstream, 0.5 chord length, much far away for the \( c_{p_{min}} \) location.

To improve the numerical solution from these RANS computations, we considered a reduced computational domain behind the trailing edge of the propulsor blade that encompassed only the region of interaction of the two vortices. The RANS solution of Yang [49] provided the initial conditions for the grid points of the reduced domain, and the boundary conditions everywhere but at the downstream end of the domain, where an extrapolation scheme was used [46]. As in the previous section, a direct numerical simulation (DNS) of the Navier Stokes equations was performed, for a set of increasingly finer grids.

Figure 15 shows a comparison of the resulting pressure coefficient, \( C_p \), along the vortex center line between the RANS computation [49] and the DNS computations for three different grids. As the gridding is refined, \( C_{p_{min}} \) converge to about -11 at a location 0.34 chord length downstream from the tip trailing edge. Another minimum of the pressure is also seen at 0.5 chord length and has value of -10.8.

![Figure 15](image1.png)

Figure 13. Effect of the strength of two vortices on the location and intensity of the minimum pressure [45].

The two co-rotating vortices periodically approach each other during the vortex merger. As they move closer, the flow in the axial direction is accelerated and results in a decreased pressure in the vortex center. Figure 16 indicates that this pressure drop is directly connected to the enhancement of the axial velocity to a maximum value by the merger.

A bubble population was allowed to propagate through the propeller flow field and the resulting dynamic cavitation inception was studied using both 3D bubble dynamics and SAP [46]. Figure 17 illustrate where the cavitation event occurs in the flow field, the bubble trajectory and size variations are plotted with the...
propulsor blade and iso-pressure surface. It is seen that the cavitation event occurs at a location very close to the experimental observation, since the bubble grow to their maximum size near a location 0.5 chord length downstream of the tip trailing edge.

![Figure 15. Pressure coefficient at various distances from the propeller blade as computed by RANS and by the direct Navier Stokes solution with an increasing number of grids [46].](image)

![Figure 16. Pressure coefficient and axial velocity at various downstream distances from the propeller blade [46].](image)

**Conclusions**

Difficulties in considering real fluid effects have led the user community to select a liquid only simple engineering definition of cavitation inception as the basis for cavitation predictions and scaling. While this has served the community very well for decades, advances in silencing and detection has made such a definition unsuitable for advanced designs.

One has then to resort to the more basic definition of cavitation as that of the explosive growth of initially microscopic nuclei in a liquid, resulting in visible bubbles (optical criterion) or in detectable emitted sound signals (sub-visual cavitation and acoustical criterion). The required simulations of the nuclei behavior in complex flow fields and turbulent structures were previously out of reach of the community and
thus were out the question. However, with the advent of desktop high speed computers and with the development of advanced computational techniques, this is now within the reach of designers who are increasingly using CFD (such as RANS) to select their designs. In this communication, we have proposed a practical method to actually conduct bubble dynamics numerical experiments as in the real flow field. This allows actual nuclei fields to interact with the computed flow field. This method, which we have successfully used with RANS solvers, a DNS solver, could be used with experimentally measured flow fields and become a design tool for cavitation avoidance.

We have shown some simulations in the body of the communication for cavitation inception. In fact, the method has also been used more recently to simulate advanced cavitation such as shown in Figure 18 and could prove with further development to be a powerful design and scaling tool. One of its major strengths is that it allows the engineer to reproduce and mimic the actual experimental procedures. For instance, both acoustical and optical criteria of cavitation inception could be measured. Concerning the acoustical criteria, the technique provides in addition to amplitude of measured signals, the number of events per second, and the spectra of the sound generated, which both could be used to simulate detection. The engineer could therefore utilize the same criteria and tools as used in the real life experiments to conduct the predictions.

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