Study of Tip Vortex Cavitation Inception Using Navier-Stokes Computation and Bubble Dynamics Model

The Rayleigh-Plesset bubble dynamics equation coupled with the bubble motion equation developed by Johnson and Hsieh was applied to study the real flow effects on the prediction of cavitation inception in tip vortex flows. A three-dimensional steady-state tip vortex flow obtained from a Reynolds-Averaged Navier-Stokes computation was used as a prescribed flow field through which the bubble was passively convected. A "window of opportunity" through which a candidate bubble must pass in order to be drawn into the tip-vortex core and cavitate was determined for different initial bubble sizes. It was found that bubbles with larger initial size can be entrained into the tip-vortex core from a larger window size and also had a higher cavitation inception number.

1 Introduction

The type of cavitation known as hydrodynamic cavitation can take many physical forms, or geometric shapes, at inception. There are two primary groups, traveling cavitation (traveling bubble; bubble ring; travelling patch) and attached cavitation (sheet; band; fixed-patch; spot). For such devices as propellers, several forms of cavitation may appear simultaneously due to the different flow characteristics on the pressure and suction sides of the blades and the quite different flow over the tip. The three-dimensional flow around the blade tips of propellers produces an additional form named tip vortex cavitation, which is related to the travelling bubble form of cavitation (Billet and Holl, 1979). Of particular interest is cavitation in the trailing vortex system since the tip vortex cavitation is associated with inboard noise generation and induced hull vibrations and in some cases is the first type of cavitation to appear. It may also be responsible for erosion problems in certain propulsive configurations such as ducted propellers.

In general, flows with cavitation are characterized by the cavitation number \( \sigma \) which is defined as

\[
\sigma = \frac{p_v - p}{1/2 \rho U^2},
\]

with \( p_v \) and \( U \) being the freestream pressure and velocity, respectively, \( \rho \) the liquid density, and \( p \) the vapor pressure at the bulk temperature. The cavitation inception number, \( \sigma_i \), describes the conditions at the onset of cavitation when approached from noncavitating flow at higher \( p_v \). Engineering predictions of cavitation inception for a single phase flow are often made by equating the cavitation inception number to the negative of local minimum pressure coefficient. The onset of the cavitation, however, has been observed to occur at pressures above and below the vapor pressure. The deviation from the ideal cavitation number is caused by some real fluid effects such as the random pressure fluctuation, bubble/flow interaction, bubble deformation, nucleus size spectra, and distribution. These real flow effects always complicate the inception process and the phenomenon is governed by intractable mathematics rich in nonlinear behavior. The complexity of the problem has led various studies to neglect one or more of the factors in play, and therefore to only investigate the influence of a limited set of parameters.

Two different approaches have been used to study the dynamics of traveling bubble cavitation inception in vortex flows. In one approach (Bovis, 1980; Chahine et al., 1995; Sarkar et al., 1995) the underlying flow is described by inviscid potential flow but important phenomena such as the modification of the vortex flow by the presence of the bubble and the shape deformation of the bubble are considered. The second approach (Johnson and Hsieh, 1966; Latorre, 1982; Ligneul and Latorre, 1989) considers real fluid effects to determine the bubble motion equation, but neglects the bubble shape deformation and modification of the flow caused by the bubble. Since the bubble/vortex interaction is significant only when the ratio of bubble size to vortex-core size is large, the second approach can offer good approximation for small ratio of bubble size to vortex core size. For tip vortex cavitation inception, the traveling bubble is usually small relative to the vortex core. The second approach is, therefore, adopted in the present study.

In the second approach, the gas nucleus is assumed to remain spherical during volume variation so that a relatively simple bubble dynamics model known as the Rayleigh-Plesset equation (Pelesset, 1948) can be applied to determine the bubble volume variation with time. Since tip vortex cavitation inception is related to the traveling bubble form of cavitation, further assumption of discrete and noninteracting nuclei can be made for computing bubble motion. This assumption allows the adoption of the nucleus motion equation of Johnson and Hsieh (1966) coupled with the Rayleigh-Plesset equation to determine the bubble trajectory in the tip vortex flow and the influence of the nucleus size and distribution on cavitation inception. Latorre (1982) successfully applied these two equations to deduce noise emission in the tip-vortex cavitation. However, the tip vortex flow field was given by using a simple Rankine vortex model. To realistically simulate the bubble moving around the tip vortex flow, the tip-vortex flow over a finite-span hydrofoil calculated from the Reynolds-Averaged Navier-Stokes (RANS) equations with a turbulence model is used as the prescribed flow field in the present study. Although the random pressure fluctuations are expected to influence the bubble trajectory, the bubble tra-
jectory in the mean flow will still be a good approximation of the ensemble-averaged trajectory in the actual time-varying turbulent flow.

2 Numerical Method

2.1 Navier-Stokes Computation. In the present study, the three-dimensional incompressible Navier-Stokes flow solver, INS3D-UP, developed by Rogers et al. (1991) is adopted to solve the tip vortex flow over a finite-span hydrofoil. The INS3D-UP code is based on the artificial-compressibility method in which a time derivative of pressure is added to the continuity equation to couple it with the momentum equations. As a consequence, a hyperbolic system of equations is formed and can be solved using a time-marching scheme. The spatial differencing of the convective terms uses a fifth-order accurate flux-difference splitting based on Roe’s method (1981). A second-order central differencing is used for the viscous terms. The resulting system of algebraic equations is solved by a Gauss-Seidel line-relaxation method in which several line-relaxation sweeps through the computational domain are performed before the solution is updated at the new pseudo-time step. This method can be marched in pseudo-time to reach a steady-state solution where a divergence-free velocity field is obtained. In the present study, the steady-state solution is acquired when the maximum divergence of velocity is less than 10⁻³.

The INS3D-UP code is also accompanied by the Baldwin-Barth one-equation turbulence model (Baldwin and Barth, 1990) which is derived from a simplified form of the standard k-ε equation. This model is not only simpler than the two-equation model, but also eliminates the need to define the turbulent mixing length which is required in the Baldwin-Lomax algebraic model.

Since the multiblock scheme (one block for suction side and one block for pressure side) is used in the present study, there are two types of boundaries where conditions have to be specified: 1) the physical boundaries, such as inflow, outflow, far field, and solid surfaces; and 2) the block-interface boundaries across which all flow quantities must be continuous. For the physical boundaries, freestream velocity and pressure are specified at the far-field boundary and the inflow boundary while the first-order extrapolation for all variables is used at the outflow boundary. On the solid hydrofoil surface, no-slip flow and zero normal pressure gradient conditions are used. At the root section a symmetric boundary condition is applied. For the block-interface boundaries, a semi-implicit method of passing the boundary conditions between blocks can be easily accomplished by updating the velocities and pressure at the block-interface after each Gauss-Seidel line-relaxation sweep of a block. The next sweep through the other block would utilize the updated values at the common block interface.

2.2 Bubble Dynamics Model. By considering all forces acting on a spherical particle with radius \( R \), the dimensional vector form of the motion equation (Maxey and Riley, 1983) is written as

\[
\rho_b V_b \frac{d \mathbf{U}_b}{dt} = \mathbf{F}_{\text{b}}(\mathbf{V}_b - \mathbf{V}_s) - \mathbf{F}_{\text{drag}} - \mathbf{F}_{\text{bush}}
\]

where parameters with the subscript \( b \) are related to the bubble/particle and those without the subscript \( b \) are related to the carrying fluid. \( \mathbf{V}_b \) and \( \mathbf{V}_s \) are the bubble volume and projected area, which are equal to \( 4/3\pi R^3 \) and \( \pi R^2 \), respectively. The bubble drag coefficient \( C_D \) in Eq. (2) can be determined by using the empirical equation of Haberman and Morton (1953):

\[
C_D = \frac{24}{Re_b} (1 + 0.197 Re_b^{0.65} + 2.6 \times 10^{-4} Re_b^{0.89})
\]

where the bubble Reynolds number is defined as

\[
Re_b = \frac{2R|\dot{U} - \dot{U}_s|}{\nu}
\]

The first term on the right-hand side of Eq. (2) is the buoyant force. The second term is due to the pressure gradient in the fluid surrounding the particle. The third term is the drag force. The fourth term is the force to accelerate the virtual "added mass" of the particle relative to the ambient fluid. The last term, called the Basset term, takes into account the effect of the deviation in flow pattern from steady state. Equation (2), however, does not include the lift force which is caused by the particle spin.

Since Eq. (2) is developed to describe the motion of a solid particle in the flow, one may need to adapt the equation for a gas bubble. For a gas bubble the mass of the gas inside the bubble can be neglected since it is small compared to the added mass of the fluid. To describe the motion of a gas bubble, however, one may not only need to take into account the forces due to the bubble volume variation. Johnson and Hsieh (1966) added an additional term to consider the bubble volume variation but the Basset term was neglected. An analysis by Morrison and Stewart (1976) shows that the Basset term depends on the time rate of change of the relative velocity. For flows in which the frequency of the oscillatory motion of the carrier fluid is small the Basset term can be neglected. Since the prescribed flow field for carrying the bubble was computed from the RANS equations, random turbulent fluctuation has been eliminated.

Although the bubble will still experience unsteady velocities when the bubble travels through the steady flow field in time, the rate of change of the relative velocity is small compared to other forces. Since the bubble will be released at the local water velocity, the Basset term is neglected in the present study.

By adding an additional term to consider the bubble volume variation, neglecting the Basset term, and using the Euler equation for relating the local pressure gradient to the acceleration of the fluid elements, Eq. (2) can be rearranged as

\[
\frac{d \mathbf{U}_b}{dt} = -2g - \frac{3}{\rho} \nabla p + \frac{3}{4R} \frac{C_D}{Re_b} (\dot{U} - \dot{U}_s) |\dot{U} - \dot{U}_s| + \frac{3}{2} \frac{dR}{dt}
\]

Equation (5) is the equation given by Johnson and Hsieh (1966) except that they omitted the buoyant term. The last term of the right-hand side in Eq. (5) is the additional term related to the bubble volume variation.

To determine the bubble volume variation with time, the Rayleigh-Plesset equation given by Plesset (1948) is used:

\[
R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 = \frac{1}{\rho} \left[ p_s + p \left( \frac{R_0}{R} \right)^{2k} - p - \frac{2S}{R} - \frac{4\mu}{R} \frac{dR}{dt} \right]
\]
where $p_o$ is the initial gas pressure inside the bubble with $k$ the polytropic gas constant ($k = 1$ for isothermal behavior), $p$ is the ambient pressure local to the bubble, and $S$ is the surface tension. The Rayleigh-Plesset equation is derived with the assumptions of liquid incompressibility and spherical bubble variation. The bubble size grows principally in response to gaseous expansion and to an increase in the vapor mass within the bubble. The pressure within the bubble is considered to be the sum of gas and vapor pressure with the assumptions of polytropic gas and equilibrium vaporization process.

To determine the bubble motion and its volume variation, the Runge-Kutta fourth-order scheme is applied to integrate Eqs. (5) and (6) through time. The effect of the underlying flow is to produce a prescribed pressure field through which the bubble is passively convected. The flow field computed from the RANS equations is applied to provide the ambient pressure and velocities local to the bubble. The numerical solution of the RANS equations, however, can only offer the result at grid points. To obtain the values of pressure and velocities at an any specified point $(x, y, z)$ in the computational domain, one needs to interpolate the values from the solution at the grid points. The three-dimensional linear interpolation scheme as described by Thompson et al. (1985) is applied in the present study. Computations of the bubble trajectory were conducted using the RANS solution on the primary grid (1.2 million grid points) and a higher resolution grid (2.2 million grid points). Since only slight differences were found for the bubble trajectories produced by the two grids, the primary grid was used for all results presented here.

3 Results and Discussion

3.1 3D Steady-State Tip Vortex Flow. In the present study, a rectangular hydrofoil having a NACA 0015 cross section with a chord length $c = 0.52$ m and an aspect ratio (based on semi-span) $Ar = 3$ is considered. For the Navier-Stokes computation, the present study uses an H-H type grid which contains $135 \times 91 \times 61$ for suction side and $135 \times 91 \times 41$ for pressure side in the streamwise, spanwise and normal direction respectively. In this grid, the tip vortex core includes at least 17 grid points in the crosswise direction and 28 grid points in the spanwise direction. An extensive grid-independence study and validation of numerical solution with experimental data were conducted by Hsiao and Pauley (1998). It was shown that the tip vortex flow was well predicted in the near-field region, where cavitation inception occurs although over-dissipative and dissipative errors were found far downstream. The comparison of the tangential and axial velocity components across the tip-vortex core between the numerical solution and experimental data (McAlister and Takahashi, 1991) at $x/c = 1.1$ for $Re = 1.5 \times 10^6$ and $\alpha = 12^\circ$ is shown in Fig. 1. The tip vortex can be visualized by creating particle traces near the tip region as shown in Fig. 2.

3.2 Bubble Trajectory. For computation of bubble dynamics and motion equations, all water properties, the density ($\rho = 998$ kg/m$^3$), viscosity ($\mu = 0.001$ kg/ms), surface tension ($S = 0.0728$ N/m), and the vapor pressure ($p_v = 2337$ Pa), are defined at $20^\circ$C. According to previous experimental data collected by Billet (1984), the nucleus size measured in different water tunnels and in the ocean ranges from several microns to 200 $\mu$m. The initial bubble size used in the present study will therefore be within this range. Since the size of the tip-vortex core in the near-field region is about 30 mm for the current case (measured from peak to peak in Fig. 1.a), the ratio of bubble size to vortex-core size is very small and neglecting bubble/vortex interaction is justified.

In a practical situation, various bubble sizes can be located at any position in the free stream. When determining the conditions for incipient cavitation, however, we are interested only in the initial positions which will lead the bubble along its trajectory to the minimum pressure region. In studying the influence of initial bubble size on its trajectory for a two-dimensional half body, Johnson and Hsich (1966) found a "screening" process in which the pressure gradient field ahead of the leading edge expelled larger sized bubbles and only allowed smaller sized bubbles to approach the body surface. Since the minimum pressure was located on the body surface, only bubbles smaller than a certain size can encounter the minimum pressure in the flow field. Such a screening process, however, may not occur in the three-dimensional tip-vortex flow. Maines and Arndt (1993) suggested that there was a "window" through which a candidate bubble must pass in order to be drawn into the tip-vortex core and cavitate but they did not measure enough data to have a reliable statistical mean. Furthermore, it is expected that the "window of opportunity" will be different sizes for different initial bubble sizes.

To better understand the window of opportunity, bubbles with different initial sizes are released ahead of the hydrofoil. In the present study, a streamwise location ahead of the hydrofoil leading edge is specified for releasing bubbles (see Fig. 3). Ideally, this streamwise location should be located at infinity ahead of the hydrofoil. Far upstream of the hydrofoil, however, the pressure gradient produced by the hydrofoil has negligible effect on the bubble trajectory and bubble size. The bubble will remain its original size and virtually follow the streamline if no external force (such as the buoyant force) is present. To save computational time, the initial streamwise location is chosen at a place where the pressure gradient can be neglected. To determine this streamwise location a bubble ($R_0 = 100$ $\mu$m) is released along a known streamline far upstream of the hydrofoil leading edge. It was found that for $x/c = -0.1$ the bubble retains its original size and virtually follows the streamline. It should be noted that the buoyant force is not included in this calculation.
For a specified initial bubble size, the window of opportunity is determined by releasing bubbles at prescribed matrix points on the $y-z$ plane at $x/c = -0.1$. Two different initial bubble sizes, $R_0 = 50$ and $100 \mu m$, are applied to investigate the influence of the initial bubble size on the window of opportunity. Usually, these prescribed matrix points are established based on trial and error. Inversely integrating the streamline from the minimum pressure location in the tip-vortex core back to the bubble release plane can offer approximate matrix points. Once a release point is found to lead the bubble to the tip-vortex core, other matrix points can be established by arranging them adjacent to that release point. More than 100 points are used to release bubbles in each case. All computations are made with a cavitation number $\sigma = 3.2$ at which no bubble will grow unstably through its trajectory. Each released bubble is tracked and the minimum pressure which the bubble experiences along its trajectory is recorded at the release point. It should be pointed out that if the bubble is entrained into the tip vortex, the bubble will encounter the minimum pressure along its trajectory in the tip vortex. Otherwise, the bubble may encounter its minimum pressure on the hydrofoil surface. Only release points which cause the bubble to be entrained to the tip vortex will therefore be considered here. Another consideration is that some release points will lead bubbles to collide with the hydrofoil due to the local pressure gradient. It is assumed that after collision the bubble trajectory will remain tangent to the hydrofoil surface as it moves past the hydrofoil. After collision, however, most bubbles will stop moving when they encounter a local minimum pressure on the hydrofoil surface because the fluid velocity is very small near the surface.

To produce the window of opportunity, matrix points which lead the bubble to encounter a negative minimum pressure coefficient along its trajectory larger than 3.0 are plotted as a contour with the encountered minimum pressure coefficient. Figures 4 and 5 show the contours on the release plane for two different initial bubble sizes. Matrix points whose streamlines pass
through the negative pressure coefficient larger than 3.0 are also plotted as contours shown in Fig. 6. In each figure the right boundary of the contour is formed by those release points which cause the bubble to collide with the hydrofoil surface. It is seen that the shape of the window is strip-like. The window is always located below the tip leading edge, \((y, z) = (1.56, 0.0)\) m, and on the pressure side. As the initial bubble size is decreased, the window location and size will approach that of the streamline in which the bubble size can be assumed infinitely small. Comparison of Figs. 4–6 shows that larger bubbles are entrained into the tip vortex core from a larger initial area. In other words, bubbles with larger initial size have more opportunities to enter the tip vortex core and cavitate. This result is in contrast to the “screen” process found for two-dimensional flows by Johnson and Hsieh (1966). In their study larger sized bubbles were expelled by the adverse pressure gradient field ahead of the leading edge did not prevent larger sized bubbles from entering the tip vortex. The entrainment of large bubbles into the tip vortex was also observed by Maines and Arndt (1993).

For infinitesimal bubble size, the nucleus concentration within the tip vortex core is the same as the incoming flow condition since the bubble actually follows the streamline. By comparison of Figs. 4–6, it can be seen that the window of opportunity increases with increasing bubble size. Since the region of the tip vortex core with the negative pressure coefficient larger than 3.0 is the same for each case, it is concluded that for a finite bubble size the nucleus concentration within the tip vortex core is higher than the upstream conditions. The ratio of nucleus concentration in the tip vortex core to the upstream incoming flow conditions increases with increasing bubble size.

3.3 Cavitation Inception. Before the cavitation inception number can be determined from the present bubble dynamics model, a consistent and adequate definition of the cavitation inception event is needed. Although many different definitions of cavitation inception have been suggested and applied by researchers, there are practical complications in determining the actual cavitation event. Using a scientific definition, cavitation inception can be defined as gaseous or vaporous cavitation. Rapid growth of a microbubble that remains stable is called gaseous cavitation while unstable growth is referred to as vaporous cavitation. From an engineering definition, cavitation inception is determined indirectly through visual or acoustic techniques, in which case a notion of events per unit time is often required. Cavitation inception is defined to occur when the measurement detects events above a predefined threshold. Another definition given in many laboratory studies (Ling et al., 1982) is actually directed towards gaseous traveling bubble dynamics. The cavitation inception event is usually defined by bubble growth to a threshold size.

Based on the different definitions described above, three different criteria will be used in the present study to determine the cavitation inception number. The first criterion given by Shen and Gowing (1986) assumes that the cavitation inception occurs when the bubble has grown to 10 times its original size. The cavitation inception number determined by this criterion is designated as \(\sigma_1\). Instead of growing to 10 times its original size, the second criterion defines a certain size which can be detected through visual techniques in the experiment. In the present study, this size is set to 1 mm to determine \(\sigma_2\). The third criterion corresponds to the stability of the numerical solution. For unstable bubble growth in the present numerical solution, the bubble size will become negative when the bubble reduces its size after passing the minimum pressure location. By decreasing the cavitation number in a series of simulations, the first cavitation number which causes the bubble to grow unstably is identified as \(\sigma_2\).

It is known from experimental studies that the nucleus size spectrum is a determinant of the cavitation inception number. To numerically study the effect of nucleus size, cavitation inception is predicted for three different initial bubble sizes \((R_0 = 50, 100, 200 \mu m)\). From the previous section, we know that different initial bubble sizes will influence the bubble trajectory. Bubbles with different initial sizes are therefore released at different positions so that they all pass through the minimum

![Fig. 6](image-url)
pressure location in the tip-vortex core. The bubble trajectories for different $R_0$ are shown in Fig. 7. It can be seen that bubbles with different initial sizes released at different positions are all entrained into the tip vortex and downstream have almost the same trajectory. For each case, the cavitation inception number is determined using the three different cavitation inception criteria. Although each definition of cavitation inception results in a different cavitation number, the effect on the bubble trajectory is negligible as long as the variation in $\sigma$ is not very large. The cavitation inception numbers determined by applying the three different criteria are summarized in Table 1 for different sized nuclei.

It is seen that different definitions of the cavitation inception event lead to slightly different cavitation inception numbers. No matter which definition is applied, however, the cavitation inception number always increases with increasing initial bubble size. The cavitation inception numbers for the three different $R_0$ studied in the present study are all smaller than the negative minimum pressure coefficient ($-C_{pt_{min}} = 3.23$) in the tip-vortex core. Due to the bubble response time and brief passage of the bubble through the minimum pressure region, the minimum pressure is below the vapor pressure when cavitation inception occurs. The pressure field which bubbles experience along their trajectories for different $R_0$ is plotted in Fig. 8. It is seen that bubbles experience almost the same pressure field after crossing over the tip. In each case the bubble experiences two pressure peaks along its trajectory. The first peak is smaller and is encountered when the bubble crosses over the hydrofoil tip. The second peak is the minimum pressure in the tip-vortex core. It is also observed that delayed response of the bubble causes the location of the maximum bubble size ($x = 0.517$ m) to occur downstream of the minimum pressure location ($x = 0.503$ m).

To demonstrate the influence of the initial bubble size on the bubble growth rate near the cavitation location, the bubble size variations of each $R_0$ along their trajectories for different cavitation numbers are plotted in Figs. 9(a–c). In each figure three different cavitation numbers are applied to produce an increase in bubble size of approximately 4, 11, and 23 times its original size. Although the bubble returns to its original size at lower cavitation numbers, the numerical solution is terminated due to the unstable recovery rate which causes the bubble size to be negative. A reasonable explanation for this result is that the bubble collapses after passing the minimum pressure location. Oscillation of the bubble size is observed when the bubble recovers from its maximum size at higher cavitation numbers. A stable and damping variation of the bubble size, however, is found after the bubble passes the minimum pressure location. Comparison of Figs. 9(a–c) also shows that as the initial bubble size is decreased, the bubble growth rate near the cavitation location becomes more sensitive to the cavitation number, i.e., for smaller initial bubble size a slight change in the cavitation number may cause a stable bubble growth to become unstable when the cavitation number is close to $\sigma$. It is also found that the bubble response time for adjusting its size to the pressure field is shorter for smaller $R_0$.

### 4 Conclusions

A window of opportunity through which a candidate bubble must pass in order to be drawn into the tip-vortex core and

<table>
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<tr>
<th>$R_0$</th>
<th>50 $\mu$m</th>
<th>100 $\mu$m</th>
<th>200 $\mu$m</th>
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<td>3.11</td>
<td>3.15</td>
<td>3.16</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>3.10</td>
<td>3.15</td>
<td>3.22</td>
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<tr>
<td>$\sigma_3$</td>
<td>3.11</td>
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<td>3.22</td>
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Fig. 8 The bubble size variation along the bubble trajectory at different cavitation numbers for a) $R_0 = 50 \mu$m, b) $R_0 = 100 \mu$m c) $R_0 = 200 \mu$m

Fig. 7 The pressure field which bubbles experience along their trajectory for different $R_0$.
Cavitation was determined for different initial bubble sizes. Bubbles with larger initial size can be entrained into the tip-vortex core from a larger window size. This implies that the cavitation inception event will be easier to observe for a flow with larger nuclei.

Three different criteria were applied to determine the cavitation inception number in the present study. It was found that different definitions of the cavitation inception event lead to different cavitation inception numbers. No matter which definition was applied, however, the cavitation inception number was smaller than the negative minimum pressure coefficient in the tip-vortex core and increased with increasing initial bubble size.

The bubble growth rate near the cavitation location became more sensitive to the cavitation number as the initial bubble size was decreased, i.e., for smaller initial bubble size a slight change in the cavitation number may cause stable bubble growth to become unstable when the cavitation number is close to \( \sigma_i \). It was also found that the size of initially smaller bubbles responded more quickly to pressure changes.

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