PRESSURE FIELD GENERATED BY THE
COLLECTIVE COLLAPSE OF CAVITATION BUBBLES

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ABSTRACT

The intense erosion observed in hydrodynamic flows or hydraulic machines under cavitating conditions is more likely due to collective rather than individual bubble collapse. We present in this paper a theory in which the interaction between the bubbles of a cloud is taken into account in computing the individual bubble behavior and the pressure field generated. The results obtained with the numerical code developed show that the influence of the other bubbles in the cloud on the collapse of a particular bubble is to reduce the driving pressure during most of the collapse time, thus delaying the implosion; and then to dramatically increase this pressure producing a violent end of the collapse. The pressures released are then orders of magnitude higher than with an isolated bubble.

RESUME

L'érosion intense observée dans les écoulements hydrodynamiques ou les machines hydrauliques soumis à la cavitation est probablement dûe au collapse collectif plutôt qu'au collapse individuel des bulles. Nous présentons dans ce papier une théorie qui tient compte de l'interaction entre les bulles d'un "nuage" pour la determination du comportement individuel des bulles, et le champ de pression généré. Les résultats numériques obtenus montrent que l'influence des autres bulles du nuage sur le collapse d'une bulle particulière est de réduire la pression motrice durant environ-toute la durée du collapse, freinant ainsi l'implosion, et ensuite d'augmenter considérablement cette pression engendrant une violente fin de collapse. Cet effet augmente avec le nombre de bulles interagissantes. Les pressions émises sont un on plusieurs ordres de grandeur superieures au cas d'un bulle isolée.
INTRODUCTION

In hydrodynamic flows and hydraulic machines microcavities preexisting in the fluid can grow in low pressure regions to sizable bubbles which collapse when convected to high pressure regions, or following a transient increase in the system pressure. The violent implosion of these cavities is accompanied with undesirable effects such as loss of performance, erosion damage, emitted noise or dangerous pressure surges. There have been extensive studies on the dynamics of this phenomenon, however, most have concentrated on individual spherical cavities and increasingly sophisticated theories have been developed. Less attention has been given to the more realistic, but too complex, situation of a field of interacting bubbles. However, the concerted action of collapsing bubbles is known to be responsible for the high level of erosion often observed in cavitating installations. The existence of a collective behavior is often obvious in ultrasonic fields and hydrodynamics flows [1-3], and the most impressive example of the strength of such a phenomenon is the bending of the trailing edge of foils [3,4] which cannot be explained by individual bubble collapse.

To solve the problem in its generality, one has to write the equations of motion of the two-phase medium constituted by the bubbles and the surrounding liquid, taking into account the bubble motion and deformation. This task is far from simple and earlier studies consisted of two main simplified approaches. Van Wijngaarden [2] considered a uniform, unidimensional, low void fraction layer of spherically identical bubbles, and derived the equations of motion neglecting convective and dissipative effects. March and Hansson [1,2,5] considered a dense unidimensional (spherical, cylindrical, or planar) cluster, and treated the motion of its surface as the propagation of a unidimensional shock wave which annihilates any vapor bubbles on its path. This approach is very interesting in determining the global behavior of a cluster of bubbles. However, in its present form, it is incapable of calculating accurately the pressure field, (the obtained pressures are infinite at the end of the collapse), since it does not allow the bubbles to contain noncondensables. In addition, the main physical assumption (presence of a shock wave dividing the space in two regions one containing bubbles which do not sense the pressure variations until a later time stage, and another one where all bubbles have collapsed) is valid only for relatively high void fractions. The case of a spherical single cavity of the same size as the whole cloud is the perfect extreme example of the domain of validity of this approach.

In this paper we will consider the case of a cloud of low void fraction in an infinite medium. The theory we have developed to study the interaction of two collapsing bubbles [6,7], or more generally nonspherical bubbles [8], is extended to the collective collapse of a multibubble cloud system. We will describe the asymptotic theory [9], and present for a few particular configurations of bubble distribution the individual bubble growth and collapse, as well as the pressures generated. Two classical functions of imposed pressures at infinity are considered: a sudden pressure jump and a finite duration pressure drop.

ASYMPTOTIC THEORY

We will describe in this paragraph a perturbation theory for the case of a cloud of low void fraction in an infinite medium. Provided that the characteristic size of a bubble, $r_0$, is small compared to its characteristic distance from its neighbors, $l_0$, we can assume that interactions are weak enough so that, to the first order of approximation, and in absence of relative velocity with the surrounding fluid each of the individual bubbles reacts to the local pressure variations spherically, as if isolated. With this assumption we can consider, with no further restrictions, the behavior of a given distribution of $N$ bubbles in an initially known volume; mutual bubble interactions, individual bubble motions and deformations are taken into account in the higher orders of approximation.

Since the problem possesses two different geometrical scales, $r_0$ and $l_0$, we will decompose it into two sub-problems: an "inner" and an "outer" problem. The "outer problem" is that considered when the reference length is set to be $l_0$. 
This problem is concerned with the macrobehavior of the cloud, and the bubbles appear in it only as singularities. The "inner problem" is that considered when the lengths are normalized by $r_{bo}$. The solution of this problem applies to the microscale of the cloud, or to the vicinity of the individual bubble of center $B_i$. The presence of the other bubbles, all considered to be at infinity in the "inner problem", is sensed only by means of the matching condition with the "outer problem". That is to say, physically the boundary conditions at infinity for the "inner problem" are obtained, at each order of approximation, by the asymptotic behavior of the outer solution in the vicinity of $B_i$. Mathematically, one has to match term by term the inner expansion of the outer solution with the outer expansion of the inner solution, using the same asymptotic sequence in the two expansions.

First Order Approximation

The determination of the flow field and the dynamics of any of the individual bubbles, $B_i$, is accessible once the boundary conditions at infinity in the corresponding "inner region" are known. In the absence of a slip velocity between the considered bubble and the surrounding fluid (when interactions are neglected) the only boundary condition at infinity is the imposed pressure variation $P_{oo}(t)$. The "inner problem" is therefore spherically symmetrical and its solution is given by the well-known Rayleigh-Plesset equation. With the assumption that the liquid is inviscid and incompressible this equation can be written as follows:

$$ a_0 \ddot{a}_0 + \frac{3}{2} \dot{a}_0 = -P_{oo}(t) + \frac{P_g}{a_0} (a_{-3k} - 1) + W_e^{-1}(1 - a_{-1}) $$

In this equation, where the superscript $1$ is omitted for convenience, $a_i(t)$ is the radius of the bubble $B_i$ normalized by $r_{bo}$. The times are normalized by the Rayleigh time,

$$ T_o = \frac{r_{bo}}{\rho_s / (P_o - P_v)}^{1/2} $$

$$ \overline{P}_{so} = \frac{P_g}{a_0} (P_o - P_v) $$

$$ \overline{P}_{oo}(t) = \frac{(P_{oo}(t) - P_o)}{(P_o - P_v)} $$

where $P_o$ is the initial pressure, and $P_v$ the vapor pressure. The Weber number is related to the surface tension, $\gamma$, $(P_o - P_v)$ and $r_{bo}$ by:

$$ W_e = \frac{r_{bo} \cdot (P_o - P_v)}{2 \gamma} $$

The noncondensable gas pressure inside the bubble, $P_g$, is assumed to have a polytropic behavior, $P_g a_{ok} = cte$, where $k$ is the polytropic coefficient ($1 < k < cp/cv$).

For a given $P_{oo}(t)$, equation (1) can be solved for the variations of the bubble radius, $a_i(t)$. This allows the subsequent determination of the pressure field around the bubble $B_i$, of center $B_i$, by the use of:

$$ P_o(B_i, r, t) = \overline{P}_{oo}(t) + r^{-1}(2 a_0 \dot{a}_0^2 + a_0^2 \ddot{a}_0) - a_0 \dot{a}_0^2/2r^4 $$

where $r$ is the distance between $B_i$ and a given point $M$ in the fluid.

Interactions

When interactions cannot be neglected, still assuming that an "inner region" enclosing the bubble $B_i$ can be defined, the boundary conditions at infinity can be much more complex than in the preceding paragraph. First, the macroscale pressure in the cloud at $B_i$, $P(B_i, t)$, can be very different from the imposed far field pressure $P_{oo}(t)$. Second, a relative velocity between the bubble and the surrounding fluid, $U(B_i, r, t)$ can exist causing the bubble to be nonspherical. Both $P$ and $U$ can be determined only by solving the equations of motion of the two-phase
medium. Such an approach will be considered in future work. Here we will limit ourselves to a small perturbation theory whose interest will be to give the behavior of the solution when the perturbation grows continuously. In that case \( P(B^i, t) \), which is the driving pressure for the collapse of the bubble \( B^i \), is only a perturbation of the imposed far field pressure, \( P(t) \), and \( U(B^i, r, t) \) is a perturbation of the spherical velocity due to the bubble volume variation.

If we assume that the liquid flow is irrotational, we can define a velocity potential for the macroscale ("outer problem"), \( \phi(B^i, t) \), and a velocity potential for the microscale ("inner problem"), \( \phi^i(B^i, t) \) such as:

\[
\Delta \phi = \Delta \phi^i = 0
\]

The matching condition between these two potentials expresses the \( \infty \)-infinity conditions for \( \phi^i \), and replaces the conditions on \( P(B^i, t) \) and \( U(B^i, r, t) \). This is written:

\[
\lim_{B^i M \to \infty} \phi^i(B^i M, t) = 0 \quad + \quad \varepsilon \phi_1(b^i, t) + \varepsilon^2 \phi_2(b^i, t) + \ldots
\]

Since bubble interactions vanish when \( \varepsilon \) goes to zero, in the absence of relative velocity between the surrounding fluid and the bubbles in the cloud, \( \phi_0 \equiv 0 \).

\( \psi_1, \psi_2, \ldots \) are the contributions of the whole cloud to the boundary condition at infinity for the inner problem (1). Using the results obtained with the interaction of two bubbles and the property of addition of potential flows, this condition can be written:

\[
-\lim_{r \to \infty} \phi^i(B^i M, t) = \sum_{j=1}^{N} \left[ \left( \begin{array}{c} j \n 2 \\ \nu_0 \end{array} \right) \right] \left( \varepsilon q_0^j + \varepsilon^2 q_1^j + \varepsilon^3 q_2^j + \ldots \right) ^n + \left( \begin{array}{c} j \n 2 \\ \nu_0 \end{array} \right)^2 \left( \varepsilon q_0^j + \varepsilon^3 q_3^j + \ldots \right) \frac{r \cos q_0^j}{r^3} + \left( \begin{array}{c} j \n 2 \\ \nu_0 \end{array} \right)^3 \left( \varepsilon q_0^j + \ldots \right) \frac{r^2 p_2(\cos q_0^j) + \ldots}{r^3} \right]
\]

where the superscript \((j)\) denotes quantities corresponding to the other bubbles, \( B^j \). \( d_{0i} \) is the initial distance between the bubble centers \( B^i \) and \( B^j \). \( \theta_{0i} \) is the angle \( M B^i M \) and \( r \) is the distance \( B^i M \), where \( M \) is a field point in the fluid (see Figure 1). \( P_n(\cos \theta) \) is the Legendre polynomial of order \( n \) and argument \( \cos \theta \).

\( q_0^j \) is the correction of order \( \varepsilon^n \) of the strength, \( q_0^j = \Delta \phi(a_j^0)^2 \), of the source representing the first-approximation spherical oscillations of the bubble \( B^j \). Expressed in physical terms (velocities, pressures), the boundary condition (9) states that the first order correction, \( (\varepsilon) \), to the nonperturbed spherical behavior, \( a_i^0(t) \), of the bubble \( B^i \) is a spherical modification of the collapse driving pressure. This would introduce, as for two bubbles, a spherical correction \( a_i^1(t) \) of the variations \( a_i^0(t) \). At the following order, \( (\varepsilon^2) \), a second correction of the uniform pressure appears, as well as a uniform velocity field accounting for a slip velocity between the bubble and the surrounding fluid. Again, as in the two-bubble case, this induces a spherical correction, \( a_i^2(t) \), and a non-spherical correction \( f_i^1(t) \) \( \cos \theta_{0i} \), where \( \theta_{0i} \) is a direction to be compounded from all the \( \theta_{0j} \). Things become more complex at the order of expansion \( \varepsilon^3 \), where in addition to the uniform pressure and velocity corrections, \( a_i^3(t) \) and \( f_i^2(t) \) \( \cos \theta_{0i} \), a velocity gradient is to be accounted for, to generate a non-spherical correction, \( g_i(t) \) \( (3 \cos^2 \theta_{0i} - 1)/2 \).
The equation of the surface of the bubble $B_i$ can be written in the form:

$$R(\theta^{ij}, t) = a_0^i(t) + \varepsilon a_1^i(t) + \varepsilon^2 \left[ a_2^i(t) + f_2^i(t) \cdot \cos \theta^{ij} \right] +$$

$$+ \varepsilon^3 \left[ a_3^i(t) + f_3^i(t) \cos \theta^{ij} + g_3^i(t) P_2 (\cos \theta^{ij}) \right] + \ldots, \quad (10)$$

where $a_0^i(t)$ is given by the Rayleigh-Plesset equation (1), while the other corrections are obtained by solving the following differential equations in which the superscripts (i) are omitted for convenience:

$$\begin{align*}
a_0 \ddot{a}_1 + 3a_0 \dot{a}_1 + a_1 F_0(a_0, W_e, P_{g_0}, K) &= \Sigma_j - \frac{P_0}{\kappa_{ij}^0} q_{0j}^0, \\
a_0 \ddot{a}_2 + 3a_0 \dot{a}_2 + a_2 F_1(a_0, a_1, W_e, P_{g_0}, K) &= \Sigma_j - \frac{P_0}{\kappa_{ij}^1} q_{1j}^1, \\
a_0 \ddot{a}_3 + 3a_0 \dot{a}_3 + a_3 F_2(a_0, a_1, a_2, W_e, P_{g_0}, K) &= \Sigma_j - \frac{P_0}{\kappa_{ij}^2} q_{2j}^2, \\
a_0 \ddot{a}_4 + 3a_0 \dot{a}_4 &= \Sigma_j - 3 \left( \frac{\kappa_{ij}^0}{\kappa_{ij}^4} \right)^2 \left( a_{0j} q_{0j}^1 + a_{0j} q_{0j}^1 \right), \\
a_0 \ddot{a}_5 + 3a_0 \dot{a}_5 + 3F_3(a_0, a_1, j) \dot{d}_j &= \Sigma_j - 3 \left( \frac{\kappa_{ij}^0}{\kappa_{ij}^4} \right)^2 \left( a_{0j} q_{0j}^1 + a_{0j} q_{0j}^1 + F q_{0j}^1 \right), \\
a_0 \ddot{a}_6 + 3a_0 \dot{a}_6 - 6/\kappa_{ij}^0 a_0^2 \dot{g}_j &= \Sigma_j - 5 \left( \frac{\kappa_{ij}^0}{\kappa_{ij}^4} \right)^3 \left( a_{0j} q_{0j}^1 + 2a_{0j} a_{0j} q_{0j}^1 \right)
\end{align*}$$

(11)

In these equations $F_0, F_1, F_2, F_3$ are known functions depending on the physical constants, $W_e$ and $P_{g_0}$, and on the calculated preceding orders of approximation. The deformations $f_2, f_3$ of the bubble $B_i$ and the motion of its center toward $B_j$: $\ell_2, \ell_3$; have been replaced by $d_2, d_3$ which indicate the total motion of the point $E_1$ (Figure 1).

$$\begin{align*}
d_2 &= \dot{f}_2 - \dot{\ell}_2, \\
d_3 &= \dot{f}_3 - \dot{\ell}_3
\end{align*}$$

(12)
When all the initial radii of the bubbles in the cloud are identical, these right-hand sides are obtained by multiplying the two-bubble case right-hand sides by one of the geometrical constants $c_1, c_2, c_3$:

$$c_1 = \sum_j \left( \frac{L_j}{L_0} \right),$$

$$c_2 = \sum_j \left( \frac{L_j}{L_0} \right)^2 \cdot \cos \theta_{ij},$$

$$c_3 = \sum_j \left( \frac{L_j}{L_0} \right)^3 \cdot P_2(\cos \theta_{ij}).$$

We can now compute the behavior of $B_i$ by solving the obtained differential equations using a multi-Runge-Kutta procedure. The behavior of the whole cloud can then be obtained. This appears at first to be a very long task. However, there exists for each of the equations, what we will call a unit-solution from which the real solution can be immediately deduced for any other initial bubble size. Indeed, this is evident for $a_o'(t)$ where one can show that if $a_0(t)$ is the non-dimensional solution for a bubble of unit initial radius, the solution $a_o'$, for a bubble of normalized initial radius $\lambda$ is such that:

$$a_o'(\lambda t) = \lambda \cdot a_o(t)$$

The comparison of equations (11) and those obtained in the case of two-bubbles shows that the $N$ bubbles in the cloud other than $B_i$ can be replaced by a unique bubble of strength, $q_{ig}$, located at $C_i$, a distance $\ell_{ig}$ from $B_i$ in the direction defined by the angle $\theta_{ig} = 0_{ij}$. As this equivalent bubble should induce the same pressures and velocities as defined by (9), its characteristic are obtained by the equations:

$$q_{ig} = \sum_{j=1}^{N} q_{nj} / L_0,$$

$$\ell_{ig} = \sum_{j=1}^{N} \ell_{nj} / L_0.$$

where $\ell_{ig}$ and $\ell_{ij}$ are respectively unit vectors of the directions $B_iC_i$ and $B_iB_j$ (Figure 1), and $n$ is the order of approximation. These equations define the angle $\theta_{ig}$, and the direction in which $d_i(t)$ is measured (equations (10) and (12)).

Pressure Field

Once the "inner problems" are solved, the nondimensional outer potential, $\varphi$, can be written:

$$\varphi(M,t) = -\sum_i \left[ \varphi_{oi} + \frac{q_i}{r_i} + \varepsilon \frac{q_i}{r_i^2} + \varepsilon^2 \left( \frac{q_i}{r_i^3} - \frac{h_i}{r_i^2} \cos \theta_{ig} \right) + O(\varepsilon^3) \right],$$

where bars denote nondimensional "outer" quantities, and tildes nondimensional "inner" quantities.

$$\bar{\varphi} = \phi - \varepsilon T/r_b^2, \quad \bar{q_i} = q_i - \varepsilon T/r_b^3, \quad \bar{r_i} = r_i/r_b.$$

$T$ is the characteristic time of the bubble collapse and $r_b^2$ is the distance between a field point $M$ and $B_i$. The Bernoulli equation enables one to calculate $P$ using (17). In the nondimensional form we have:

$$\bar{p}(M,t) = \frac{p(M,t) - P_\infty(t)}{\Delta p} = -\frac{3}{2} \frac{\partial \phi}{\partial t} - \frac{1}{2} \varepsilon^2 \left| \nabla \phi \right|^2.$$
\( \Delta p \) is the amplitude of the pressure driving the collapse and \( t = t/T \), where

\[
T = r_{b_0} \sqrt{\rho/\Delta p} \quad (20)
\]

In the following, we will consider as an illustration a uniform field of bubbles; any bubble has the same geometrical position relative to the others, and thus the same behavior. The general expression (8) simplifies considerably to become:

\[
p(M,t) = (e^{q_0} + e^{q_1} + e^{q_2} + e^{q_3}) \sum_i \left( \frac{1}{r_i^4} \right) + \sum_i \left( \frac{\cos^2 \theta_i}{r_i^{12}} \right) - \sum_i \left( \frac{1}{r_i^4} \right) + 0(e^3) \quad (21)
\]

In this expression, the two first summations are geometrical constants similar to \( c_1, c_2, (4) \). The last one is more complex, but is more easily calculated when written as follows:

\[
\mathbf{\nabla} \sum_i \left( \frac{1}{r_i^4} \right) = \sum_i \left( \frac{-1}{r_i^{12}} \right) e^{i\mathbf{m}} \quad (22)
\]

where \( e^{i\mathbf{m}} \) is the unit vector of the direction \( B^i M \). If one knows the direction, \( \mathbf{MV_0} \), of the velocity at \( M \), at the first order of approximation, and if \( \alpha V_0 \) is the angle \( B^i \mathbf{MV_0} \) (Figure 1), then:

\[
\mathbf{\nabla} \sum_i \left( \frac{1}{r_i^4} \right) = \sum_i \left( \frac{\cos \theta \alpha V_0}{r_i^{12}} \right) \quad (23)
\]

**NUMERICAL ILLUSTRATION: SPHERICAL SHELL OF BUBBLES**

We consider a distribution of bubbles centered on the surface of a sphere, and we admit that each of the bubbles has the same position relative to the others. In this case the expression (21) is valid, and the numerical computation time is reduced. As examples we will consider the bubble behavior and the pressure generated for two types of ambient pressure time functions. In both cases the bubbles are at equilibrium with the ambient pressure, \( P_0 \), at \( t = 0 \). Then, in case A, the ambient pressure jumps to a new constant value, \( P_0 + \Delta P \), at the following instants. In the second case B, the imposed pressure drop first to a constant value, \( P_0 - \Delta P \), keeps this value until \( t = \Delta T \) and then comes up again to the initial value \( P_0 \) (Figure 2). As illustration we will consider the pressures generated a) in the center of the sphere; b) at the location of the bubble \( B^i \) if it were removed and c) in a point outside the cloud at a distance \( r_{b_0} \) from \( B^i \). We will compare the results with the isolated bubble case.

Knowing the initial bubble configuration and thus \( c_1, c_2, \) and \( c_3 \) the relation between the cloud radius, \( R \), and \( \ell_0 \) is: \( R = \frac{1}{2} \ell_0 \frac{c_1}{c_2} \). In the cloud center, position (a), the three summations in (21) have respectively the values \( N/R, N/R \) and 0). In position (b) these values are \( (c_1, c_2, \) and \( c_3) \), and at a distance \( r_{b_0} \) from \( B^i \) the values are approximated by \( (c_1 + e^{-1}, c_2 - e^{-2}, (c_2 + e^{-2})^2) \).

**Bubble Dynamics**

Various spherically symmetrical cloud configurations were investigated numerically. In Figure 3, the results of five different computation for a sudden jump in the imposed ambient pressure, are compared, expansions being conducted up to \( e^3 \). The ratio, \( e = r_{b_0}/\ell_0 \), was kept constant and at a value of 0.05. The cases of two, three and twelve bubbles are presented together with that of an isolated bubble. The fifth case is an intermediate situation between the configurations of three and twelve bubbles. This case is arbitrary and is only determined by the choice of \( c_1, c_2, \) and \( c_3 \). In each case the variation with time of the distance, \( B^i \mathbf{E}^i \), between the extreme point on a bubble \( \mathbf{E}^i \), and its initial center, \( \mathbf{B}^i \), is chosen to represent the bubble dynamics. Taking the bubble collapse in an unbounded
fluid as reference, it is easy to see from Figure 3 how increasing the number of bubbles changes the dynamics of the one studied. We can observe first that, during the early slow phase of the implosion process, the collapse is significantly delayed. At any given nondimensional time the distance between $B_i$ and $E_i$ (and simultaneously the bubble characteristic size) is greater when the number, $N$, of interacting bubbles increases. Then, in the final phase of the implosion the tendency is reversed: the phenomenon speeds up and, in a shorter total implosion time, the final velocities of the motion are higher when $N$ increases. The effect is explained in the following paragraph.

Figure 4 shows the behavior of the bubbles in the case of a pressure variation of type B (Figure 1). The cases of an isolated bubble and two, three, five and twelve bubbles are investigated again, and the variations of $B_iE_i$ with time are represented. The ratio $\epsilon$, and the duration $\Delta T$, of the pressure drop are kept constant and at the particular values of 0.1 and 0.8 respectively. Here, as in the preceding figure, noticeable changes can be observed when the degree of interaction increases. First, the growth is slowed down and retarded in comparison with the isolated case. Then, the collapse is accelerated and as a result the total implosion time decreases with an increase in the number of bubbles, $N$. While for $N = 2$, the total implosion time is greater than that of an isolated bubble, for $N = 12$ the time is significantly smaller. As we will see below this
acceleration of the collapse makes the generated pressures at the end of the collapse higher than for the single bubble case.

Figure 5 compares for the same cloud configuration (twelve bubble, \( \varepsilon = 0.1 \)) the bubble behavior for three values of the duration, \( \Delta T \), of the pressure drop. The greater \( \Delta T \) is, the longer the bubble is allowed to grow. As a result the maximum size it attains is bigger, but its lifetime is smaller. Thus, the resulting collapse is much stronger.

FIGURE 4 - MOTION OF THE BUBBLE WALL TOWARD THE MULTIBUBBLE CLOUD CENTER

\[ W_e = 100, \quad P_{12} = 0.53, \quad K = 1.4, \quad \varepsilon = 0.1. \]

DURATION OF THE PRESSURE DROP \( \Delta T = 0.8 \).

Generated Pressures

To examine the observations made above let us consider the variations of the pressure generated at a distance \( \ell_0 \) by the collapse of a bubble of initial radius \( r_{b0} \) in an infinite medium, following both a sudden pressure jump in the ambient pressure (Figure 2A) and a finite time pressure drop (Figure 2B). As we can see from Figure 6, the perturbation pressure i.e., the difference between the pressure at \( \ell_0 \) and the far-field pressure, is negative for \( t < 0.75 \). This observation is important in the sense that a fictitious bubble placed at the distance \( \ell_0 \) from this spherical bubble will sense a less important and more gradual increase in the surrounding pressure. In the considered case, instead of a sudden nondimensional jump of the pressure from 0 to 1, \( P \) surges only to 0.84, then rises slowly, not attaining 1 until \( t > 0.75 \). This would affect the bubble dynamics exactly as observed in Figure 3, namely a less violent start of the collapse. As a result, we find at the end of this process a larger bubble than would be observed in an infinite medium. This, added to the fact that in the later stages (\( t \approx 0.75 \)) the driving pressure increases up to 2.25 times the far-field pressure, makes the subsequent end of collapse much more violent.

The same type of reasoning can be applied to the case of a finite-time pressure drop. As we can see in Figure 7, in the first time period, \( \Delta T \), the pressure sensed at a distance \( \ell_0 \) from the bubble center, \( B_0 \), is higher than the imposed one. As a result a second fictitious bubble placed at this distance from \( B_0 \) would have a slower growth during this first period, \( \Delta T \). This phenomena is however reversed in the second phase as an expansion wave is generated by the growing bubble \( B_0 \). In the third and last phase a compression wave increases the driving pressure for collapse making this one more intense.
FIGURE 5 - INFLUENCE OF THE PRESSURE DROP DURATION ON THE BUBBLE WALL MOTION TOWARD THE CLOUD CENTER, \( W_e = 100, P_{so} = 0.53, K = 1.4, \varepsilon = 0.1, N = 12 \)

FIGURE 6 - PRESSURE VARIATION VERSUS TIME AT A DISTANCE \( l_o \) FROM AN ISOLATED SPHERICAL BUBBLE, AMBIENT PRESSURE JUMP \( P_{so} = 0.1, W_e = 100, \varepsilon = r_o/l_o = 0.33 \)

FIGURE 7 - PRESSURE VARIATION VERSUS TIME AT A DISTANCE \( l_o \), FROM AN ISOLATED SPHERICAL BUBBLE, FINITE TIME AMBIENT PRESSURE DROP \( P_{so} = 0.53, W_e = 100, \varepsilon = r_o/l_o = 0.2, \Delta T = 0.8 \)
In figure 8, we can see an example of the pressures generated during the bubble history at two locations: a) the center of the cloud and b) the center of one bubble, Bi, in its absence. These pressures are compared with those generated during the growth and collapse of an isolated bubble at a distance equal to the spherical cloud radius. We have selected the case of a finite time, ∆T = 0.6, pressure drop. The same observations made while interpreting figure 7, can be repeated here where they are seen to be much more accentuated. After the imposed ambient pressure increases, the nondimensional pressures generated by the twelve bubbles cloud are first positive, then a pressure expansion period is observed for 1.9 < t < 3.4, followed by a high pressure surge at the end of the collapse. The corresponding bubble radius variation with time is that represented in Figure 5 (12 bubbles ∆T = 0.6).

Figure 9 is a collection of the results obtained in several cases studied. The maximum nondimensional pressure generated during the cloud collapse are represented versus the number of bubbles in the cloud. The cumulative effect is obvious since the values obtained vary in a several orders of magnitude range. The numbers represented should not be considered accurate since other scales for times, pressures and lengths are needed at the end of the collapse. Instead, they are presented here to give an indication of how tremendous pressures can be generated with an increasing number of interacting bubbles, and to give an idea of the trend of this increase. In this figure, the maximum pressures are given...
at the cloud center, C, at \( B^\dagger \) when it is removed and at a distance \( r_{bo} \) from the center of one of the bubbles in the cloud.

These results show the important role played by the gas content of the bubbles which was neglected in \([1,2]\). Increasing \( P_{go} \) from 0.1 to 0.2 has dramatically reduced the generated pressures. This comes mainly from the fact that the cushioning effect of the gas reduces significantly the velocities attained at the end of the implosion.

Another very interesting observation from figure 9 is that the imposed pressure variation \( B \) (pressure drop of finite duration followed by a recompression), moves the maximum pressures generated at the end of the collapse toward much lower values in comparison with the pressure jump case A. This effect is not due to the apparent higher gas content in this case. Indeed, the value of \( P_g \) to consider for comparison purposes should be for all cases that at the start of the collapse when the bubble has its maximum volume. For the case of twelve bubbles for example and a pressure drop \( (\Delta T = 0.8, \ P_{go} = 0.53) \) the value of \( R_{max}/R_0 \) is 1.63 (Figure 4). Then, accounting for the gas expansion, the gas pressure at the beginning of the collapse is \( P_g = P_{go} (1.63)^{-4.2} = 0.07 \). The effective gas content is thus smaller, and since the value of \( \varepsilon \) is bigger (0.2 instead of 0.1) the observed pressure drop is intrinsically related to the imposed pressure function. The pressure attenuation observed is explained by the initial influence of the cumulative effect on the bubble behavior in the cloud which is not the same at the start of the growth or at the collapse (Figure 6,7). In the classical pressure jump case the initial cumulative effect is to prevent the bubble size from being small when the collapse pressure surge starts. Conversely the initial cumulative effect in the second type of imposed ambient pressure (case B) is to reduce the size of the bubble when the collapse pressure surge occurs.

Finally Figure 10 shows the influence of the duration of the pressure drop on the maximum pressures generated. The previous type of reasoning when applied to the gas pressure leads us to believe that the increase of the maximum pressure with \( \Delta T \) is mainly due to a decrease in the effective initial gas content at the start of the collapse since the maximum bubble radius increases with \( \Delta T \).
This study has shown that, even for very low void fractions, collective bubble collapse can generate pressure orders of magnitude higher than those produced by single bubble collapse. This would tend to explain the observed high erosion intensities and the bending of trailing edges. The cumulative effect comes from the fact that the interaction increases the driving pressure of collapse of each individual bubble. This augments the violence of its implosion and thus the interaction with the other bubbles. Thus, each bubble ends its collapse not under the effect of a pressure of the same order as the ambient, but orders of magnitude higher. This cumulative effect would not exist if the void fraction is high enough for the cloud to behave as a single bubble. This leads us to believe that there exists a critical value for the void fraction for maximum erosion.

One major assumption of the study is that the liquid is incompressible. This assumption is valid as long as the fluid velocity does not exceed the speed of sound. For single bubble dynamics this does not usually happen until the final phase of the collapse. Here, however, two factors contribute to limit the validity of the assumption. First, the rate of implosion is higher and second, more important, the velocity of sound drops considerably when the void fraction increases. This introduces a possible serious limitation on the values of the accepted void fraction, \( \alpha \), (or on \( \varepsilon \) since \( \alpha \sim \varepsilon^3 \)) for the validity of this approach. Another limitation, of the same nature, appears in the asymptotic theory as \( \varepsilon < 1/c_1 \). This seems to fix the domain of application of the present method to values of \( \alpha \) less than \( 10^{-3} \). Both limitations would become less important if the model were modified to allow for a compressible behavior at the macroscale, in which case a delay in the propagation of the far-field pressure into the cloud would be accounted for by a finite speed of sound. In the same way, the number of bubbles interacting with \( B^2 \) would then be limited to a radius determined by the speed of sound and the period of oscillation of the bubble.

Precise experimental observations on the collapse of a cloud are rather rare [1, 4, 10]. The most detailed observations of an ultrasonic cavitation cloud was carried out by Ellis [15] as early as 1955. We have selected Figure 13 from this reference and drawn the bubble contours in the cloud for two subsequent frames during a growth phase (Figure 11a) and a collapse phase (Figure 11b). The results of these observations are considered to be quite surprising. Even for this relatively high void fraction (presumably, \( \alpha > 10^{-2} \)) a "multibubble" behavior rather than a whole cloud motion of the periphery toward the center is clearly seen. It is obvious from these pictures that, at any given time, the pressure information reaches the bubbles on the periphery as well as those in the center.
during the growth as well as the collapse. This reinforces the idea that the results of an asymptotic study like this one could be extended to higher void fraction values.

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REFERENCES


